A MONOGRAPH

ON

FULL-SCALE DYNAMIC TESTING

FOR MODE DETERMINATION

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SAN DIEGO, CALIFORNIA

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FOREWORD

This report presents another in a series of monographs devoted to stability studies in the field of structural dynamics. The series documents work conducted by Convair division of General Dynamics for the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS8-11486. D. R. Lukens is the project leader. Convair division personnel contributing to this monograph were R. L. Turney, R. L. Fefferman, J. W. Kittle, and T. E. Reed.

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1/INTRODUCTION

This monograph will introduce the concepts, techniques, and problems associated with full-scale dynamic test programs of large booster vehicles, both solid and liquid fueled. This discussion will be aimed toward the measurement of the elastic modes that can affect control system analysis. It will include definitions of various excitation systems, suspension systems, mounting systems, instrumentation, data reduction methods, problem areas, and test planning.

The need for dynamic testing can be divided into two categories. The first is the verification of existing theories as a basis for the development of new theory; the second is for acquiring information that cannot be obtained theoretically. Full-scale vehicle testing normally affords the only opportunity other than the flight itself for testing and characterizing the entire vehicle including the spacecraft. Dynamic testing has several advantages over flight testing in characterizing the vehicle. For example, the input conditions can be made simple and measured carefully so that they are well known. Wind forces are eliminated and propulsion and control system dynamics are replaced by sinusoidally varying electromagnetic shaker forces. Furthermore, far more extensive instrumentation can be used and checked carefully during test to ensure validity of results. Comparison of the cost of a series of dynamic tests and that of a vehicle flight shows that testing is relatively inexpensive insurance for flight success. Because testing can be scheduled earlier than flights without delaying the flights, considerable information can be obtained much earlier than would be possible from post-flight evaluation.

In addition it may demonstrate existence of modes of oscillation or load paths not brought out through analysis or static test. In the large complex vehicle systems that exist today there are almost endless possibilities for coupling between the motions of the various parts of the system. In most cases such coupling is observed and measured either in dynamic testing or in flight before any theory is developed to explain the phenomenon. In cases where theory indicates the possibility of such coupling, careful measurements are necessary to establish the existence and extent of such coupling. Again this can best be accomplished through testing.

Those dynamic characteristics of the launch vehicle constituting a part of the total system must be defined experimentally, as must be the spacecraft characteristics, before a theory for the dynamic behavior of the total system can be established. The motion of complex structural elements must be defined. For example the upper spider beam of the Saturn I vehicle or the engine gimbal system and thrust structure of almost any vehicle are difficult to analyze and must be defined by testing. The effect of joints and connections upon the overall vehicle must also be established.

The effect of local resonances, which may be overlooked in the formulation of a mathematical model, can be important to control system analysis. This is particularly noticeable where control sensors are located or where control forces are applied. It is of great importance to the control system that these effects be defined; considerable efforts are being expended to develop theories that are capable of describing them.

2/STATE OF THE ART

Vibration testing to establish the natural elastic modes of a vehicle originated with aircraft when airplanes began flying fast enough to experience flutter problems. The first tests were performed on cantilever surfaces with the exciter being a camshaft with a push rod attached to the surface and driven by a variable speed motor. The first instruments used for finding node lines on the surface were an engineer's fingertips or a sprinkling of fine particles that could collect at the nodes.

From this beginning the variety of excitation devices grew to include eccentric mass shakers, air shakers, hydraulic exciters, and finally the electromagnetic exciter. Measuring instruments started with fingertips and continued through paper "arrow" amplitude indicators, bulky displacement and velocity pickups, optical devices, strain gage accelerometers, and piezoelectric transducers.

The information gathered from these early tests was very meager. Only node line definition was possible in early tests. With the introduction of more and more precise instruments, definition of the complete modal response became very good.

The elastic mode definition in airplanes was required so that possibly destructive flutter characteristics of airplane surfaces could be defined.

With the introduction of long, slender missiles and the emergence of structural feedback as a major control consideration, the information obtained from vibration tests was essential to the verification of analytical data used in the stability analysis. The slope of modes at gyro locations, the frequency and damping of the modes, and their effect on the control system response characteristics were of prime interest.

As missiles and space boosters became larger, the difficulty and hazards involved in conducting a vibration measurement program increased greatly. Suspension systems became more complex, instrumentation had to perform accurately over long distances, recording systems had to record a large number of channels of information simultaneously, and exciter systems required much closer control of the output parameters.

In spite of the above problems, successful vibration measurement programs have been conducted on large missiles and space boosters. Continuing refinement of test techniques, instrumentation, and excitation equipment are providing better test information all along the line.

3/TEST SETUP

This section discusses the pretest considerations and setups that must be made. Specifically discussed are support structure, suspension systems, excitation systems, instrumentation, and data reduction systems. Various methods of accomplishing the task in each of these areas are presented along with a discussion of the advantages and disadvantages of each.

3.1 SUPPORT STRUCTURE

This section presents the various methods of supporting the test article as far as rigid support for suspension system tiedowns, excitation systems, stabilization systems, safety bands, etc. Examples of support structures are towers, launch pads, special test supports, and structural building frames. Support structures for two basic types of vibration measurement programs are discussed; namely, cantilevered and quasi-free-free. The significance of the mounting position of the test article is discussed with relation to both liquid and solid propellant vehicles.

3.1.1 CANTILEVERED TEST SUPPORT STRUCTURE. The basic requirement for a cantilivered test support structure is that it be very rigid at the cantilever point. This could be accomplished by a concrete mass securely anchored to bedrock or if the mass itself were large enough to be considered a seismic mass. If the test could be conducted on a launch pad, this structure would adequately meet the requirements of rigidity and mass.

Some type of structural arrangement along the sides of the vehicles is required for exciter, instrumentation, stabilization, and safety restraint systems. This side structure must be isolated from the vehicle so that any response of this structure does not interfere or influence the definition of the elastic modes of the test vehicle. Figure 1 illustrates these requirements.

3.1.2 FREE-FREE TEST SUPPORT STRUCTURE. The main function of the support structure for a free-free test is to provide attachment of suspension and excitation units; hence, no severe rigidity requirements are dictated. However, this structure will have some dynamic response of its own and care must be taken to isolate this motion with respect to vehicle motion. Test towers of some sort are usually utilized to accomplish this type of support for vehicles mounted vertically. Figures 2 and 3 are two such towers used in the OAO boost vehicle test and the SAD-6 vehicle tests.

Isolation of the tower motion from the test vehicle can be accomplished most readily through the suspension and stabilization systems. Some of these towers, however, may display resonant frequencies quite close to the rigid body frequencies of the suspended test vehicle. In this case great care must be taken to not excite the

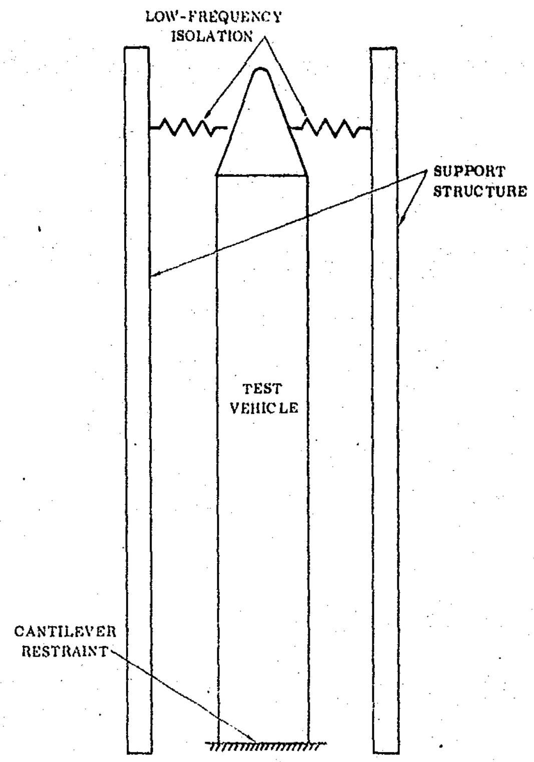


Figure 1. Low-Frequency Isolation of Support Structure and Test Vehicle

tower. When tower motion is unavoidable (e.g., wind gusts) positive vehicle motion restraints must be built into the support system so that motion above a certain pre-determined amplitude cannot be imposed on the test article.

In considering the mounting of the excitation systems to the support structure several methods may be used to isolate this equipment from the tower, such as mounting the exciters on a pendulous mass that is in turn attached to the tower. This type of isolation accomplishes two things. It isolates the tower motion from the exciters and isolates vehicle motion from the tower.

Complete filtering of these movements from each other is not possible with any isolation system. However, proper design of the exciter mounting fixtures will keep the interacting motions to a minimum. For example, a system like that shown in Figure 4 can be used for this purpose. The mass is sized both to react the exciter and vehicle forces and to display a pendulous frequency below the support structure frequencies. The mass is attached to the tower through the pendulum arms, thus isolating the interacting motions.

Figure 2. Complete OAO Test Vehicle Installed in "A" Tower at Point Loma. "B" Tower on the left contains the water storage tank. Building 3 on the right is the blockhouse.

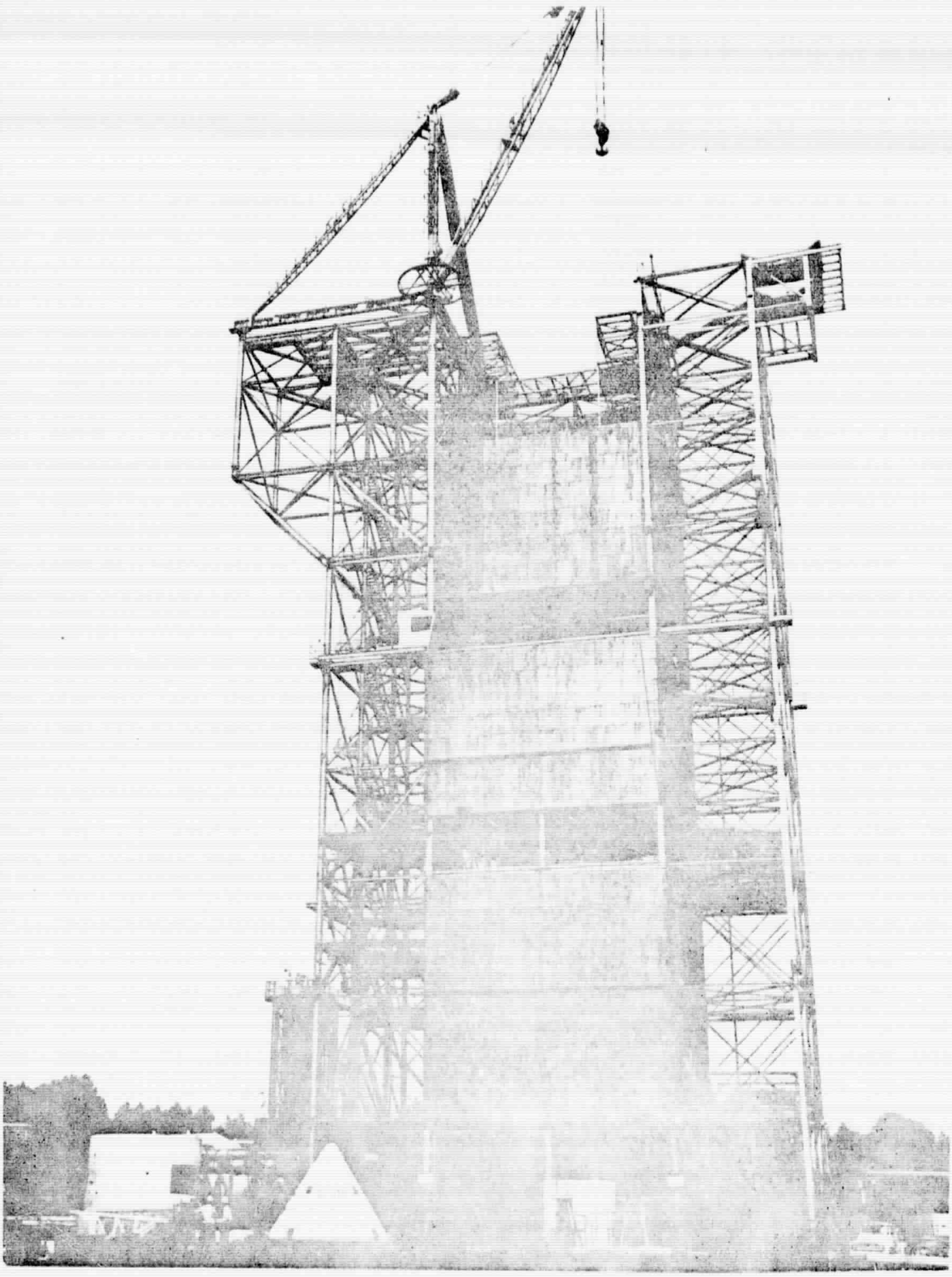


Figure 3. Dynamic Test Tower for SAD-6 Test

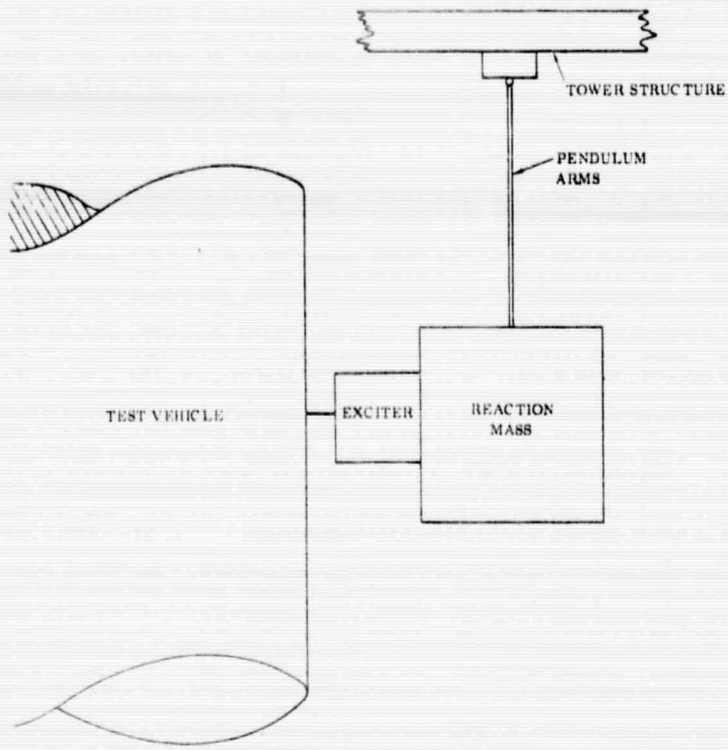


Figure 4. Pendulous Reaction Support for Excitation System

Supports for a horizontally tested vehicle are relatively simple provided the vehicle can support itself in a horizontal position. The main consideration is designing the structure so that it displays no resonance in the anticipated test range. If the vehicle will not support itself in a horizontal position, provisions will have to be made. This can be accomplished by supporting the vehicle intermittently along its length with sling supports (so-called "belly straps"); if these slings are of braided fabric, little alteration of the vehicle's mode shapes will be experienced.

3.1.3 MOUNTING POSITION

3.1.3.1 Liquid Booster. The testing program for a liquid-filled vehicle will require that it be tested in a vertical position. This is required to position the liquids internally in the vehicle during the test and also for structural strength consideration. Suspended vertically, the liquid takes up a position similar to its position in flight, thereby producing the desired effect during testing.

3.1.3.2 Solid Booster. On the other hand, solid propellant vehicles may be tested in either a horizontal or vertical position. One consideration to keep in mind when testing a horizontally supported vehicle is that if the propellant burn surface is a star-point configuration, the pull of gravity will cause a static deflection in some of the star points, thus affecting the propellant mode shapes. (This effect has been observed under test conditions.)

3.2 SUSPENSION SYSTEMS

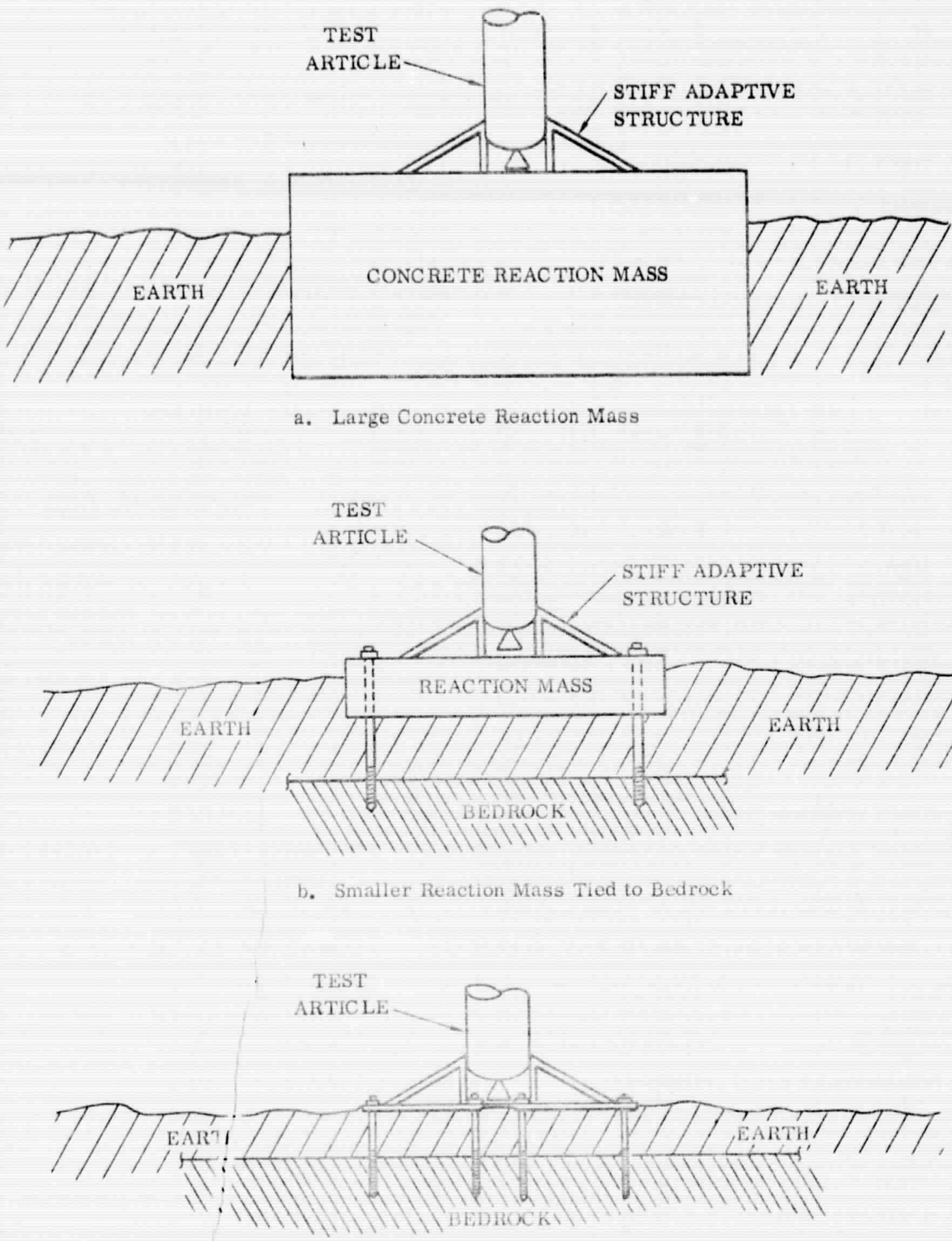
Selection of a method of simulating a desired boundary condition for a vibration measurement program is the most important single decision made at program initiation. This decision will affect many of the other considerations of the program.

This section discusses various suspension systems to accomplish both cantilevered and free-free vibration measurement programs.

3.2.1 CANTILEVERED TEST SUSPENSION SYSTEMS. The basic requirement of a cantilevered suspension system is that it display essentially zero movement at the cantilevered plane. Assuming that the test article is to be cantilevered at the base, a stiff, high load carrying member (e.g., thrust ring) must be selected as the cantilever point. A very rigid adaptor structure must then be secured between the test article cantilever structure and a large reaction mass. This reaction mass can take several forms. It can be a large, massive concrete block, a concrete and steel composite anchored by bedrock, or the earth itself by anchoring the mount directly to bedrock. Figure 5 shows these three types of reaction masses. The first natural frequency of the stiff adaptive structure should be at least 5 times greater than the highest test frequency.

For large vehicles, any of the above approaches are expensive and difficult to implement. An alternate system is one using the facilities available on a launch pad. This system affords a large reaction mass with some form of launch holddown arms. To utilize launch pad installations, consideration must be given to the flexibility of the holddown systems. By properly instrumenting this restraining structure and accounting for its response by proper removal of the effect during data analysis, good cantilevered modal information can be obtained. Another approach is to include the holddown structural flexibility in the vibration analysis and utilize the tie point between the holddown structure and the reaction mass as the cantilever point. The latter approach is usually the more desirable from the experimental standpoint while a bit more complex for the analysis since determination of reasonable spring constants for the holddown structure can prove relatively difficult. For methods of accomplishing the vibration analysis of the vehicle on the launch pad refer to the <u>Monograph on Lateral Vibration Modes</u>, GD/Convair Report GD|C-DDF65-001.

The reasoning behind a cantilever test is the verification of analytical modes. By doing a cantilevered analysis of the vehicle modes and then verifying these results with a cantilevered test of the vehicle, confidence is established as to the validity of the analytical coefficients. From this base, free-free analyses may be accomplished with confidence since this simply removes the constraints on the coefficients of the cantilevered system.



a. Large Concrete Reaction Mass

b. Smaller Reaction Mass Tied to Bedrock

c. Adaptive Structure Tied Directly to Bedrock

Figure 5. Three Reaction Masses for Cantilevered Tests

In the case of very large vehicles, (e.g., NOVA), it would be practically impossible to perform a free-free vibration test. Thus, the requirements must be met by a cantilevered test program.

3.2.2 FREE-FREE SUSPENSION SYSTEMS. When experimental results are desired to duplicate flight modes of vehicles, a free-free boundary condition simulation is necessary. Several methods of attaining this boundary condition can be implemented. Below is a discussion of four possible suspension systems for utilization with free-free vibration measurement programs.

3.2.2.1 Air Bearings. The use of a column of air to support the test vehicle has long been considered the best method for simulation of the free-free boundary condition. However, the designing and manufacturing of an operable air bearing for use on a large system was not successfully accomplished until early 1964. A more complete discussion of the design of this air bearing may be found in Reference 1.

Methods for designing air bearings have been known for many years and discussions can be found in many pneumatic design books. Figure 6 shows a cutaway view of the bearing used on the test of the OAO boost vehicle. In construction of the bearings great care must be taken to ensure surface smoothness, efficient groove patterns, and surface leveling adjustments.

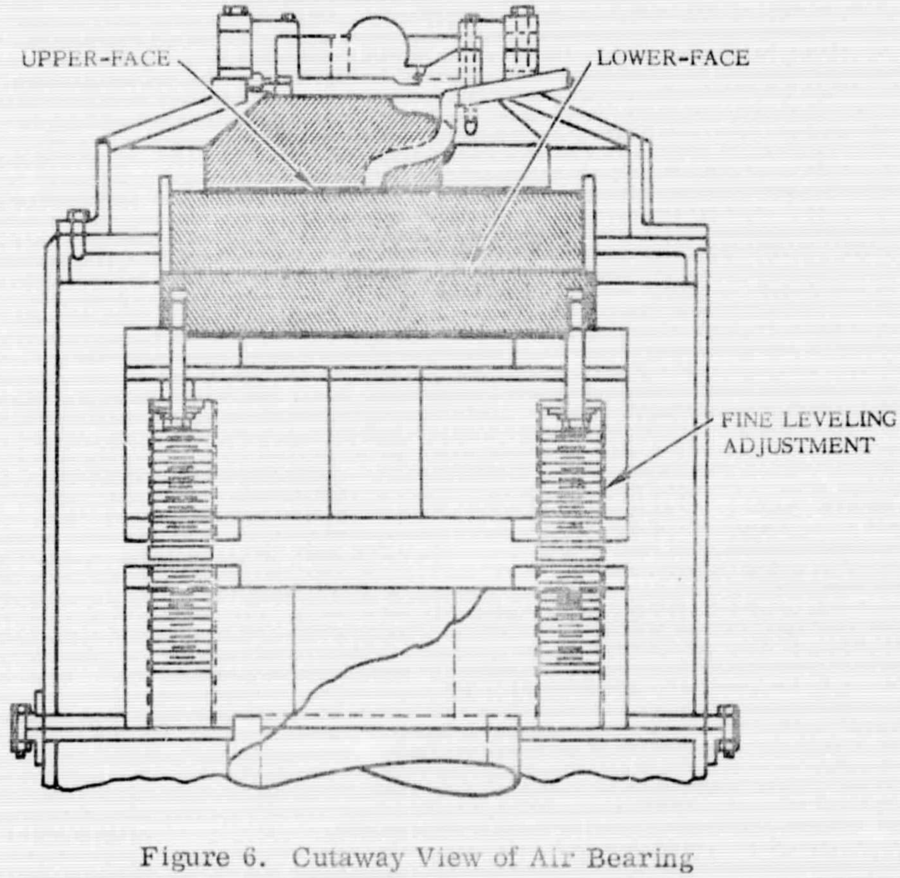


Figure 6. Cutaway View of Air Bearing

Location of the bearing on the test article must be at a good load carrying point, and, if possible, at a point of relatively small displacement during modal excitation. Such locations might be rocket engine gimbal points, main thrust ring, nozzle throat plane (solids), or any other major structural load carrying members.

It must be remembered that large modal amplitudes cannot be realized with air bearing support because small angular misalignments between the bearing faces involve pressure increases to maintain bearing separation. Since quite high pressures, up to 3000 psi, could be required for static suspension of a large vehicle, an increase of any great magnitude due to motion under dynamic test could impose limiting requirements on the pressure supply system.

Dry nitrogen is suggested as the gas to use in the air bearing since it is essentially noncorrosive and can be kept in constant supply at high pressure for long testing periods by using the boiloff properties of liquid nitrogen.

3.2.2.2 Oil Bearings. Another method that results in a free-free suspension system that produces about the same end conditions as air bearings is an oil bearing suspension system. This type of suspension basically supports the vehicle on a very thin surface of oil. Again, two surface plates are needed at each bearing, machined and polished to very smooth finishes. A plate with a concave upper surface and convex lower surface usually proves most desirable. An oil dispersion pattern in the upper surface must be designed to properly route the oil over the surface of the bearing. The oil is then forced under pressure out into the groove pattern and over the two bearing surfaces to essentially "float" the test article on a thin surface of oil, thus producing a low-friction support surface.

Attachment of the bearings to the test article will be at locations similar to those suggested in Section 3.2.2.1. Selection of a lubricating oil for use in the bearing will depend largely on pressure requirements and heating characteristics. An oil would be required that would display essentially constant viscosity throughout the heat range of the bearing.

3.2.2.3 Cable and Spring Combinations. The suspension system most used to simulate the free-free boundary condition is some type of cable suspension. This method in general provides a low-frequency support through a combination of cables and springs. This type of suspension is much more applicable to horizontal vehicle suspension than are either the air or oil bearings. It is also quite adequate for suspension of a vertically oriented test article. Reference 2 discusses various types of cables support systems and their applicability to a number of types of free-free test programs.

Figures 7 and 8 show a typical cable and spring combination suspension system utilized in the testing of the SAD-6, which was a full-scale dynamic model of SA-6. Note that this system consists of a system of cables attached to its outriggers with spring clusters located at the upper ends of the cables.

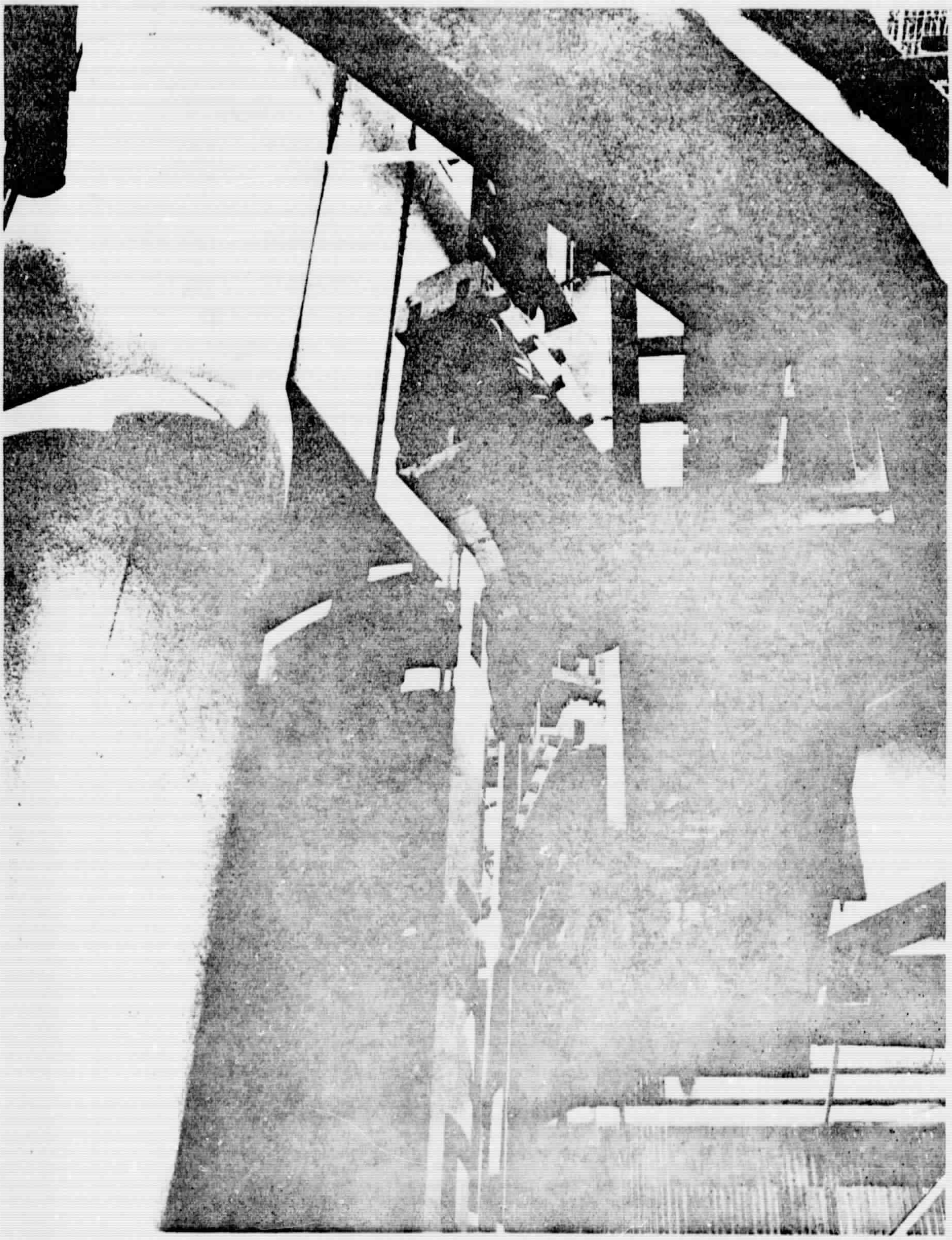


Figure 7. Cable/Spring Suspension System Used on SAD-6 Tests

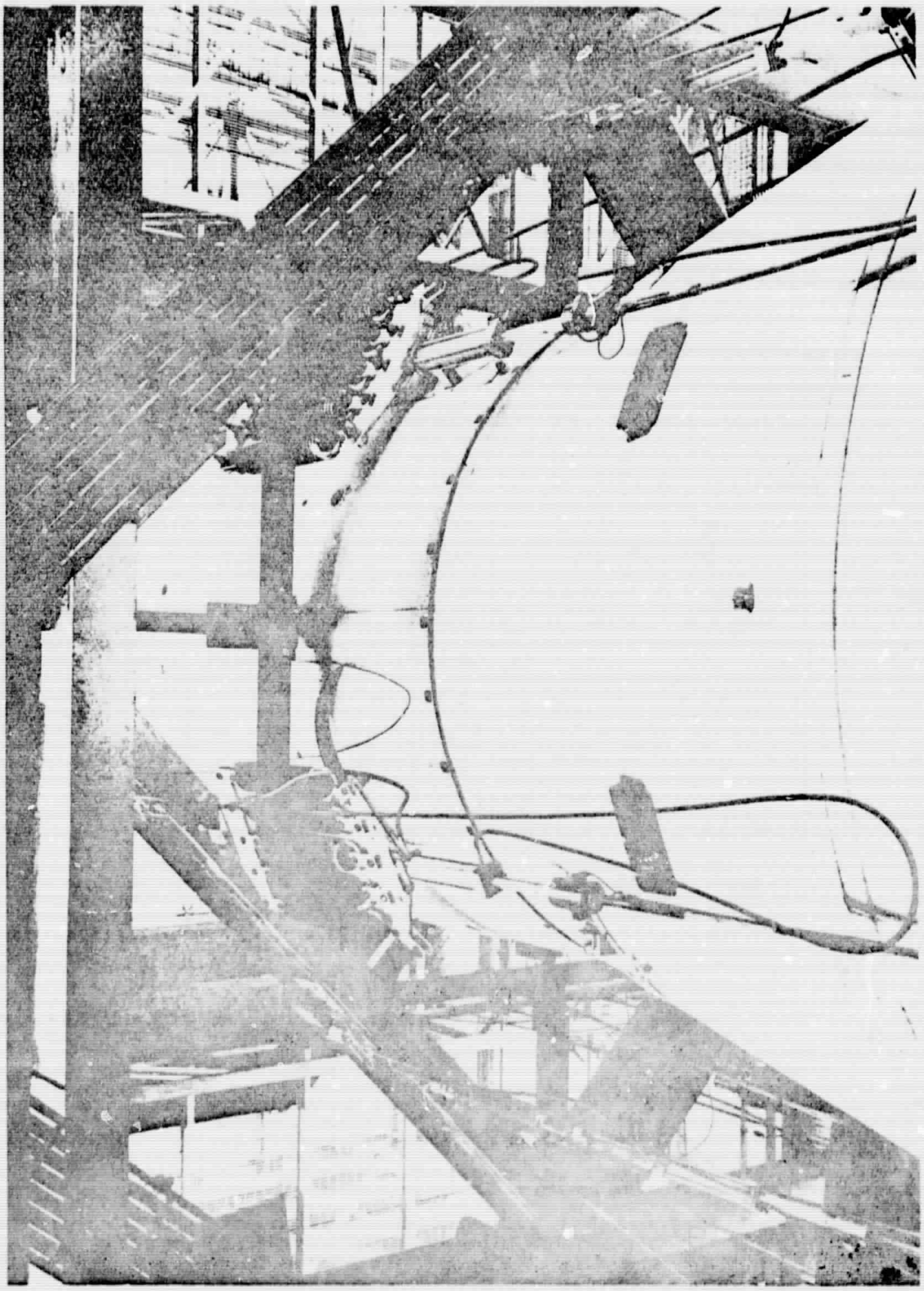


Figure 8. Typical Spring Cluster Used for S-IV Phase of SAD-6 Tests

Attachment of this type of suspension to the test article would be made at the lower end of the vehicle on a major load carrying member. In the SAD-6 suspension, hydraulic actuators were placed between the tower support structure and the spring clusters for the purpose of raising the vehicle from the ground supports to the suspended position.

In designing the springs, the first consideration is the rigid body frequencies required to effectively decouple the rigid body and elastic body modes. As a rule of thumb, the frequency of the highest rigid body mode should be a maximum of one-fifth of the lowest elastic mode of the vehicle. Also, great care must be taken in matching the spring combinations associated with the suspension cables. These must be matched so that each combination of cable/spring displays equivalent spring constants to ensure properly suspended orientation of the test article. If spring mismatch occurs, the consequences are nonvertical or horizontal alignment, coupled response of rigid body modes, and major difficulties of introducing the exciter force into the system properly.

One major drawback to this type of a suspension is that if tests are to be conducted on a vehicle for large differences in mass conditions, it is very probable that new combinations of springs will be required to produce the desired rigid body frequencies for each mass condition. Associated with this will be new preloads or large spring travel. Another consideration when using this type of suspension is that any measurements made corresponding to elastic modal damping will have superimposed parasitic damping due to the suspension system damping contribution.

3.2.2.4 Flexible Beams (Base Mounted). Another type of suspension system that can be utilized in providing an essentially free-free boundary condition is a set of base-mounted flexible beams. This system supports the test vehicle on vertical beams designed to provide flexibility in the plane of excitation. Figure 9 depicts a typical test setup utilizing this type of suspension.

In designing a suspension system of this type, the requirement that low-frequency rigid body modes be displayed by the suspended vehicle is the prime factor. This involves designing the supporting beams so that lateral response in the plane of excitation of the supported vehicle displays rigid body responses of no more than one-fifth of the lowest elastic mode frequency to be investigated. Flexibility requirements in the plane normal to the plane of excitation will be defined by the test requirements. If information is required in the plane of excitation only, then restraint of motion normal to this plane can be built into the suspension beams. If the converse is required, flexibility requirements in the normal plane will have to meet the same standards as the plane of excitation.

Attachment of this type of suspension system requires 1) a very rigid (cantilever) mount at the base of the beams and 2) attachment at the vehicle's base of a good load-carrying member or members.

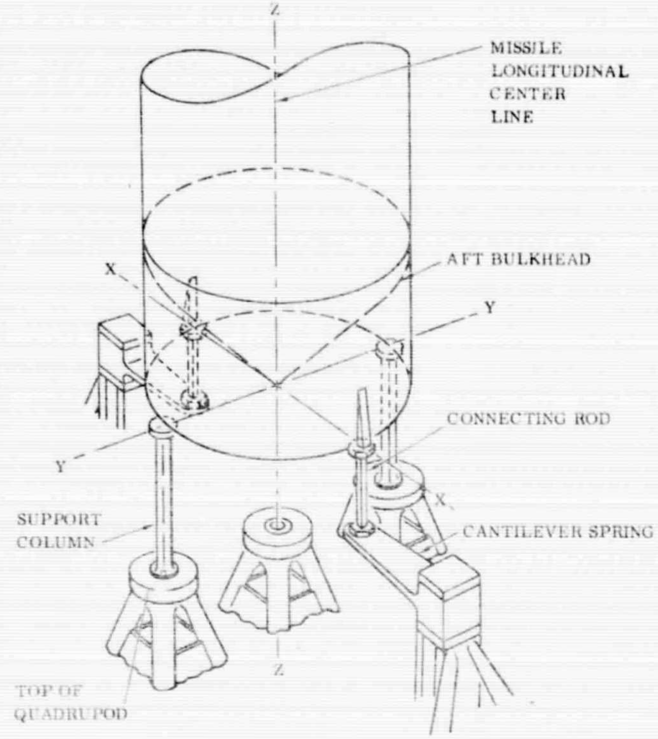


Figure 9. Flexible Beam (Base Mounted) Suspension System

This type of suspension system has limitations similar to those inherent in the cable/spring suspension. Changes of the beams to accommodate different mass conditions is a major transformation. Again, the parasitic damping caused by the flexible beams will be superimposed on the modal damping, thus making data reduction much more involved.

Table 1 is a short tabular summary of the various types of free-free suspension systems.

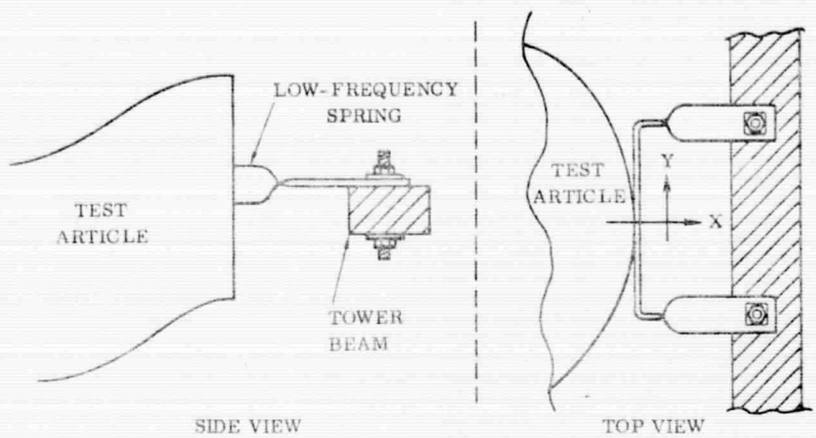
3.2.2.5 Torsional and Longitudinal Suspension Systems. The same type of suspension system may be employed when torsional mode investigations are required. However, in the case of longitudinal mode determination, the most desirable method of suspension is a cable/spring combination. A base-mounted flexible beam suspension system would be applicable to longitudinal testing but would be more difficult to design. The authors know of no instance of this being utilized for longitudinal testing.

The design criteria associated with the rigid body frequencies displayed by torsional or longitudinal suspension systems parallel that of lateral suspension systems; that is, the highest rigid body frequency must be a maximum of one-fifth the frequency of the lowest elastic mode to be investigated.

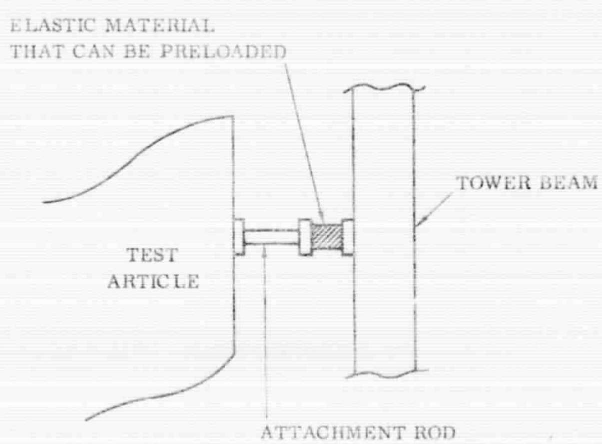
Table 1. Free-Free Suspension System Characteristics

TYPE OF SYSTEM	EASE OF CONSTRUCTING	QUALITY OF FREE-FREE SIMULATION	PARASITIC DAMPING	ADAPTABILITY TO CHANGING CONFIGURATIONS	OPERATING REQUIREMENTS	COST
Air Bearings	Very Difficult	Excellent	Almost None	Excellent	High	High
Oil Bearings	Difficult	Excellent	Most	Good	Medium	High
Spring/ Cable	Difficult	Good	Moderately Small	Fair	Low	High
Flexible Beam	Moderately Easy	Fair	Moderately Small	Fair	Low	Moderate

3.2.3 STABILIZING SYSTEMS. When the test requirements dictate a vertical mounting of the test vehicle, the suspended system is nearly always statically unstable. This is due to the attachment of the suspension system well below the center of gravity of the vehicle. Because of this instability, some sort of stabilizing system is required near the upper end of the vehicle. Examples of stabilizing systems are low-frequency springs and snubbers. Figure 10 depicts examples of snubbers and springs used as vehicle stabilizing members.



a. Example of Spring That Displays Low Resistance in the X-Plane and High Resistance in the Y-Plane



b. Example of a Snubber Stabilizing System

Figure 10. Examples of Snubber and Spring Stabilizer Systems

Probably the easiest to design and construct are the springs. These may be designed to provide low-frequency restraint in all three directions, low-frequency restraint in the plane of excitation, and high resistance in the other two planes; or any combination of these. In considering torsional investigations, low-frequency restraint is required in roll about the vehicle centerline. In designing the restraining springs, two things should be kept uppermost in the mind of the engineer; that 1) the system provides stability, and 2) the system meets the requirement of low-frequency rigid body modes dictated by the test requirements.

3.3 EXCITATION SYSTEMS

This section deals with the two most used excitation systems, hydraulic and electromagnetic. Discussions of the applications, limitations, and advantages of the two systems are presented. Location and control of excitation systems in general are also discussed.

3.3.1 ELECTROMAGNETIC EXCITERS. Electromagnetic exciters provide very good excitation to a test specimen in the frequency range of 5 to 2000 cps. Below 5 cps, the output waveform in most exciters is not sinusoidal, but sawtoothed. It is quite feasible, however, to use this type of electromagnetic excitation down to 2 cps in that the vehicle structure will act as a natural filter to any harmonic inputs. Recent developments in electromagnetic excitation equipment can provide forces up to 500 pounds and excitation down to 0.01 cps with good sinusoidal waveform. Since a control console is built specifically for this type of unit, exciter control, input and output wave shapes, and phasing are excellent within the operating frequency range. If large forces greater than 500 pounds are required for excitation, the exciter units become quite heavy and bulky. For a tower-mounted system the mass and difficulty of handling the exciter units might prohibit the use of this type of system.

With electromagnetic excitation systems, very accurate acceleration and/or displacement control can be maintained. A good sinusoidal output force signal is also inherent in this equipment along with excellent phasing properties when a multiple exciter system is required. When obtaining the damping of each mode, a negligible amount of extraneous damping is added to the system which is the flexural support of the moving portion of the shaker (the "voice" coil).

3.3.2 HYDRAULIC EXCITERS. Hydraulic exciters lend themselves to testing in the frequency range of essentially zero to 40 cps. They also are capable of producing large displacements and forces. For testing in the very low frequency range (0 to 5 cps), this type of system is more commonly available and usually adequate. For tower mounting; the hydraulic exciter is very good since only the actuator of the unit need be mounted in the tower; the actuator is not usually too massive or bulky. The power supply and hydraulic supply can be located on the ground with feeder lines running to the excitation units in the tower.

Normally these exciters are displacement controlled. It is possible to make them force or velocity controlled with modifications. The output signals from a hydraulic unit are not nearly as sinusoidal or void of harmonics as those of an electromagnetic unit. Good phasing between multiple units is quite difficult to maintain, thus requiring associated electronics with the capability to compensate for the phase shifts. Some type of disconnect apparatus should be placed between the hydraulic actuator and the test article so that modal damping measurements may be made without the parasitic damping associated with the exciter. Figure 11 shows a type of disconnect unit in the unclamped position.

Table 2 is a short tabular summary of the comparison of electromagnetic and hydraulic excitation systems.

Table 2. Electromagnetic and Hydraulic Excitation Systems

TYPE OF SYSTEM	FREQ. RANGE	BUILTIN CONTROL PARAMETERS	DISPLACE-MENT*	HANDLING PROPERTIES	COST	AUX. REACTION MASS REQD
Electro-Magnetic	(0.01 - 5)** -2000 cps	Displacement/ Acceleration	Up to 1.5 inches	Bulky	High	Little
Hydraulic	0 to 40 cps	Displacement	Up to 12 inches	Maneuverable	Moderate	Much

* Single amplitude
** 0.01 to 5 cps requires special equipment

3.3.3 NUMBER OF EXCITERS. The number of exciters to be used for mode testing depends on the following criteria: complexity of the structure, closeness of the analytical modes, mounting points available, and instrumentation and data reduction capabilities. In many tests involving simple structures, a single exciter properly located can separate the modes adequately. It may be necessary to have an exciter at an alternate location to properly separate particular modes. Complex structures quite often have closely-spaced modes that cannot be separated with a single exciter. Two approaches to this problem can be utilized; the first involves a single exciter, but results in additional instrumentation and much data reduction; the second involves multiple exciters with complex controls and instrumentation. References 3 and 4 present techniques for separating the normal modes from the total response. References 5 and 6 describe techniques for finding the force distribution necessary to produce in-phase responses when using multiple exciters.

3.3.4 EXCITER LOCATION. The location of exciter units (as to where excitation is applied to the test vehicle) depends on numerous factors. Initially, the analytical modes should be studied and exciter locations chosen where maximum amplitudes of the desired modes occur. From here, structural layouts should be consulted to determine if attachment of the exciters in these locations is possible. If not, some compromising locations are chosen where sufficient amplitude can be obtained to assure excitation of the modes. Reference 7 will provide insight to support conditions and exciter arrangements

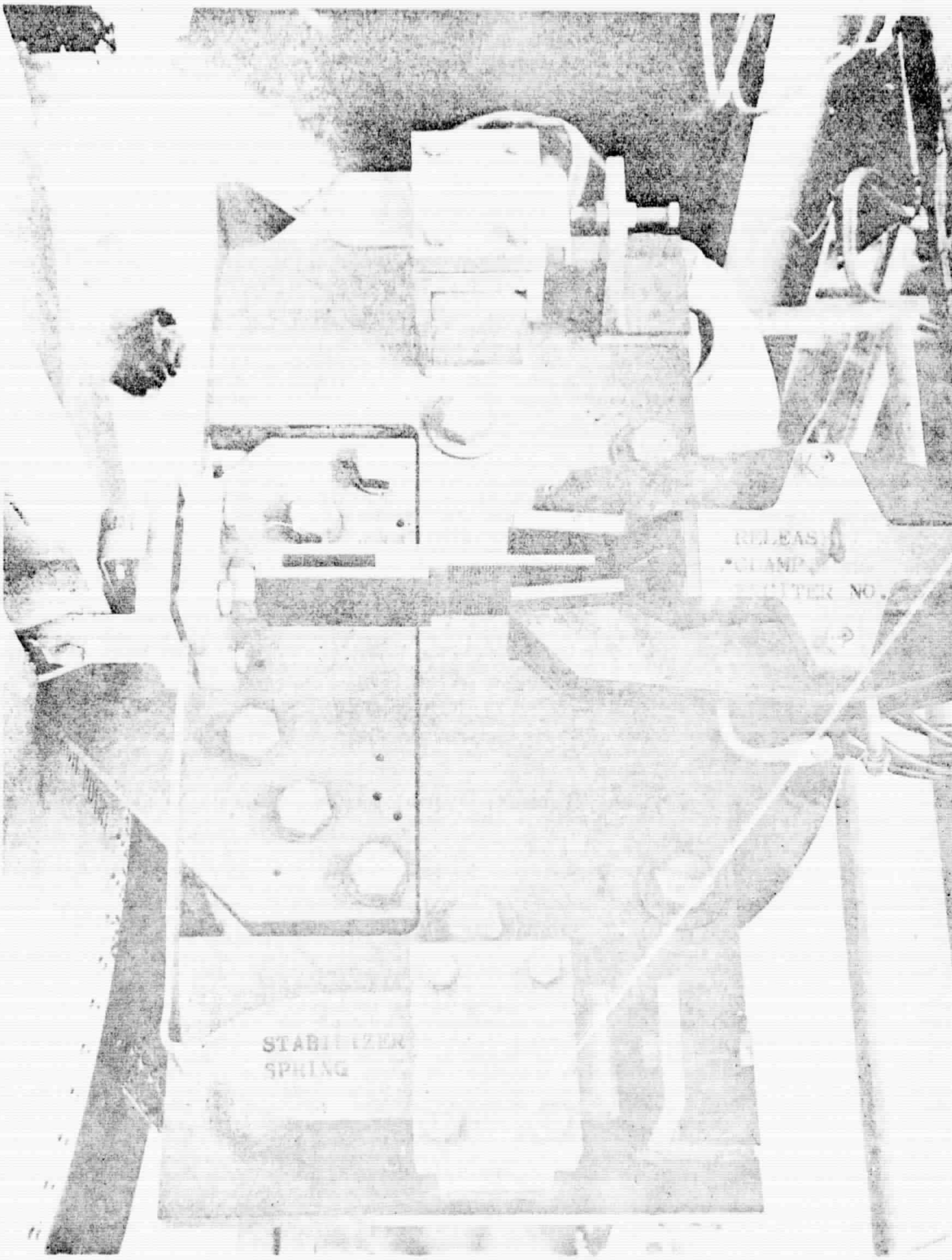
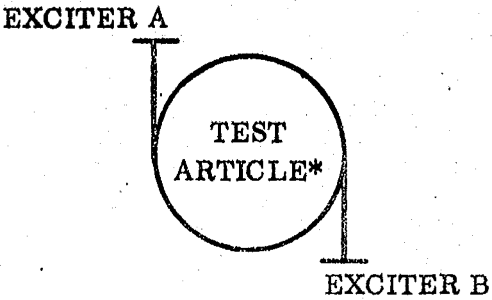


Figure 11. Hydraulic Exciter Release Clamp Mechanism (Unclamped Position)

in vibration testing. For either free-free or cantilevered testing for bending modes of a large boost vehicle, an exciter location at or near the nose is nearly always required. Depending on the complexity of the vehicle, other exciter units will be required at various locations along the vehicle, such as stage and payload interfaces, engine bells, and possible some substructures where an adequate structural tie is available.

In the case of torsional excitation, exciters should be so located that the vehicle is forced into rotary motion about its centerline. Referring to the sketch at the right, exciters A and B attached to the side of the vehicle and displaying the phase relationship indicated by the arrows as to input force will drive the test vehicle torsionally. Two sets of exciters, properly located down the vehicle, should suffice for determination of at least four torsional modes.



*Looking longitudinally down the vehicle

Longitudinal excitation is almost always applied at the base of the test article in a direction parallel to the centerline of the test vehicle. The exciter attachments should be made into a good load carrying structure such as engine gimbals, main thrust ring, etc.

3.3.5 EXCITER CONTROL PARAMETERS. As noted in the previous sections, four control parameters are available for testing. These are displacement, velocity, forces, and acceleration. Electromagnetic exciter systems have a builtin feedback system for control of displacement and/or acceleration. As a general rule, displacement is utilized as the control parameter in frequency ranges of zero to 10 cps. Above this range, acceleration as a control parameter is usually desired. Velocity control is made available by integrating the acceleration output.

In hydraulic exciter units, displacement and force are generally utilized as control parameters. Most units have a builtin displacement feedback. Force feedback usually requires that additional instrumentation be added to the basic system. Velocity control can be obtained by differentiating the displacement output.

Selection of parameters or combinations of parameters that should be used is dependent on the test requirements. In general, for the best modal measurement results, exciter forces for modal excitation should be kept small. Modes need only be driven at amplitudes large enough to ensure good definition of the deformed shape. Forcing a mode to higher amplitudes will only result in frequency and mode shape discrepancies. For large vehicle tests, it is suggested that both displacement and force limitations be incorporated in the exciter systems for vehicle protection.

3.4 INSTRUMENTATION

3.4.1 PICKUP SYSTEMS. In this section are discussed the various types of pickup systems that might be employed in a vibration measurement program. In particular, accelerometers, linear transducers, and camera or closed circuit TV systems are discussed.

3.4.1.1 Accelerometers. The most widely used vibration measurement instrument in use for description of a dynamically deformed vehicle is the accelerometer. Basically, two types of accelerometers are used, piezoelectric and strain gage.

The piezoelectric accelerometer is a crystal unit that generates a signal proportional to the acceleration of the point of attachment to the test article. This type of accelerometer system has a wide frequency range of flat response that is normally at least 2 to 15,000 cps. It also provides a large acceleration response range (0.001 to 1000 g). The accelerometer, a matched coaxial cable, and a matched amplifier (cathode follower) are usually obtained as a unit. The compactness, mass, and ease of mounting make this a very desirable accelerometer choice if no testing is anticipated in the 0 to 2 cps range. Another strong point of the piezoelectric unit is its stability to characteristics over a wide temperature range (-100° to 500° F).

When mounting most piezoelectric accelerometers, care must be taken to ensure that the coaxial cable does not vibrate. Any bending movement of the cable is apt to cause distortion in the output signal of the accelerometer, thus distorting the modal response data.

Strain-gage accelerometers are vibration measurement units that operate on the principle of a spring-mass system, the spring of which is an unbonded strain-gage bridge. The unit is usually optimumly damped in a bath fluid so that when connected to appropriate electronics it produces an output signal proportional to the acceleration of its mounted location.

This type of unit has a narrower frequency response range but will measure down to 0 cps. These units are larger and heavier than piezoelectric accelerometers and do not have large acceleration response ranges. For large vehicle testing, the size and mass of the strain-gage accelerometer is usually of no consequence. The acceleration response range, however, can be limiting and requires precision high-gain amplifiers for good performance in the low-g range if the testing will include measurement of g levels over a large range, say 0.001 to 5 g.

Mounting of this type of unit is usually quite easily accomplished and no extra care must be taken to preclude movement of the accelerometer cables as is required of the piezoelectric unit. Phasing and static calibration can easily be verified by manual orientation in the earth's gravitational field.

It must be remembered that in selecting accelerometers (either piezoelectric or strain-gage) for a modal measurement program, care must be taken to select accelerometers that display the same characteristics. Intermingling of accelerometers of different characteristics can seriously affect the resultant modal data. Reference 8 provides a complete discussion on both piezoelectric and strain-gage accelerometers.

Other types of accelerometers (servo accelerometers) exist that provide accurate measurements in the low-g range. These, however, are quite expensive per system and therefore are not applicable to multiple point modal definition.

3.4.1.2 Linear Displacement Transducers. Displacement transducers may be used in various capacities in a test program. They can be used as measurement instruments to define the vehicle mode shapes. They may be used to ascertain low-frequency rigid body responses or to determine proper orientation of the test article prior to and during excitation.

It is good practice to locate linear transducers at the top and bottom of the vehicle in both the plane of excitation and normal to the plane of excitation so that the orientation and position of the vehicle can be determined at any time during the test.

3.4.1.3 Camera and Television. Camera and television units are not generally used as data gathering units. A camera may be used as a replay instrument to substantiate data obtained through the prime units. As an example, movies may be taken of an arrow display, strategically located on the test article, to ensure the data were taken at the maximum displacement condition. Movies are also useful for documentation of the test progress from buildup to completion. Still pictures (usually Polaroid) are sometimes useful to record data displayed on oscilloscopes and not recorded in any other manner.

Closed-circuit television is quite useful in connection with vehicle safety during a remotely controlled test. TV can cover such things as major hydraulic connections, exciter alignment, pressure and heat gauges, and any other equipment that needs to be continually viewed during the test. A TV camera mounted internally in a fuel tank may be used for detecting and partially defining the motion of the fluid.

Table 3 is a short tabular summary of piezoelectric and strain-gage accelerometer characteristics.

3.4.2 RECORDING AND DISPLAY SYSTEMS. Systems that fall into this category are the units from which the principal elements of the whole program are gathered. Display systems provide the means of ensuring the test data are taken at the desired test conditions. Recording systems provide the means of permanently retaining these data for reduction and analyzation.

Table 3. Accelerometer Characteristics

TYPE OF UNIT	RESPONSE RANGE	ACCEL. RANGE	APPROXIMATE SIZE (in.^{2})	WEIGHT	AMPLIFIERS	CABLE ATTACHMENT
Piezo-electric	2-15,000 cps	0-100 g	From $\approx$ 1 to $\approx$ 0.5 in.	A Few Grams	Matched Cathode Follower	Coaxial
Strain-Gage	0-3000 cps (g-range sensitive)	0-250 g	From 3 to 1 in.	A Few Ounces	Unmatched off-the-shelf	Shielded Four-Lead

3.4.2.1 Display Systems. Several types of display systems are in wide use throughout industry; the type of system or systems required is dependent on the specific test to be accomplished.

Generally, some type of quick-look visual display of exciter output, a generation of various pickup points along the vehicle, output of any gyro packages, and phasing of exciters are required for monitoring during the test program. For comparison of various pickup locations, either direct writing records or a mode plotter system can be used. The former will record the output of several pickup units on adjacent channels of the recorder. This provides on-the-spot analysis of the comparison of pickup output for both acceleration and phasing. The direct writing recorder can also be used to monitor exciter force, velocity and displacement for control purposes, and can be utilized to monitor all positioning linear transducers for vehicle orientation.

Outputs of these units are permanent records of the actual test conditions and may be used as crosschecks against final data recordings.

The mode plotter unit is a modified oscilloscope display that will show the proportional response and phasing of the outputs of the accelerometers simultaneously on the oscilloscope screen. A complete description of the mode plotter unit utilized on the OAO boost vehicle vibration program is given in Reference 9.

This type of display unit has the advantage of simulating the mode shape on an oscilloscope screen while excitation to the vehicle is still being applied, thus allowing mode identification and amplitude peaking without removal of the excitation. To permanently record the display of the mode plotter, a Polaroid camera may be attached to the face of the oscilloscope and pictures taken of the displayed shapes.

Dual beam oscilloscopes are also used to compare gyro outputs, exciter force and displacement or velocity, and exciter phasing and forces when more than one exciter is in use, and to help in tuning modes by the displaying of Lissajous relations.

3.4.2.2 Recording Systems. Probably the most widely used data recording system is the recording oscillograph. This type of system can record many channels of data simultaneously and is relatively simple to set up. It is adaptable to either strain-gage or piezoelectric accelerometer outputs. The main disadvantage to most recording systems of this type is the time required to develop the records that must go through a process much like photographic film developing to produce readable results. There are oscillographs on the market that develop in 10 seconds after exposure to light.

Data recording on magnetic tape units has been employed in recent years for this type of testing. Here, the output signals are recorded directly on magnetic tape by use of carrier signals. Immediate replay of the recorded data is possible, thus providing quick access to any part of the record. It is also possible, by magnetic tape data recording, to play back the data through an analog computer and automatically plot the results, or possibly play back the data through filters and process the resultant filtered data on the analog computer.

By digitizing the recorded data, it is then possible to play back the recorded data through a digital computer and have the results automatically interpolated and plotted.

These last two methods of recording and analyzing data are probably the best from the accuracy standpoint. However, they are also expensive and do require programs on either the analog or digital computer to accomplish the task.

3.4.3 SPECIAL INSTRUMENTATION. For complex structures, it becomes necessary to provide additional instrumentation and/or exciter controls to obtain the data necessary to define the normal modes. The mode separation method described in Reference 3 requires that both phase lag between input force and response and the amplitude of the response be recorded over the frequency range of interest. There are instruments available that will give this phase lag directly. The Vibration Component Analyzer described in Reference 4 outputs the in-phase and quadrature components of the response. The multiple exciter system described in Reference 5 has independent exciter controls and a monitoring system that enables the operator to tune the excitation until a pure mode is being driven. This tuning procedure is performed by the feedback control system described in Reference 6.

3.5 DATA REDUCTION SYSTEMS

The problem of data reduction is of the utmost importance. No test will yield good results if the data are not properly reduced. The amount of data reduction necessary depends on the type of structure, the instrumentation used, and the reduction method employed.

3.5.1 FOR SIMPLE STRUCTURES. The modes of a simple structure will be of sufficient frequency separation that a simple data reduction method can be used. This method involves measuring the total response at each pickup location and plotting the results. Frequency can be determined by counting cycles per unit time. This will be accurate for lightly damped structures. Mode shapes are usually normalized by mathematically forcing the mode shape to where the point of maximum motion displays unit (1.0) nondimensional response. Care must be taken in reading the data to ensure adequate results. The response should be read at the same instant of time on all accelerometers. This means the point of maximum displacement is read zero to peak and then the readings of all other accelerometer outputs are read at the exact same time. It does not necessarily follow that all outputs will be peaked at this time, because of phase lags inherent in the overall system. Due to small contributions from other natural modes of the test article superimposed on the mode under investigation, phase shifts of various degrees will be noted over the vehicle.

3.5.2 FOR COMPLEX STRUCTURES. The phase shifts mentioned above become large enough to obscure the data when a complex structure is being tested. Thus a data reduction method must be employed that will separate the true modal data from the total response, or an excitation system must be used that will eliminate the response from other modes.

One method for identifying the natural modes of a complex structure is to make polar plots representing the variation of the response vector with forcing frequency. This method is presented in detail in Reference 3. Briefly, it involves plotting the response at a particular pickup location on a polar plot at fixed frequency intervals in the vicinity of a probable natural mode. The maximum spacing between the frequency points will occur at resonance. In order to determine the resonant frequency accurately, one should find the peak of the vector derivative of the response vector. The modal amplitude at this pickup location is represented by the diameter of a circle faired through the points on the plot bracketing resonance. The task of determining mode shapes for many modes soon becomes a quite lengthy process using this method.

Another method for separating the natural modes from the measured response is the phase separation technique presented in Reference 4. This method is based on the fact that the response of a damped single-degree-of-freedom system at resonance is 90 degrees out-of-phase with the force. This is called quadrature response. For a multidegree-of-freedom system, the total response at a resonant frequency consists of both in-phase and quadrature responses from that mode and also from other modes which may be excited. As mentioned previously, Reference 4 describes an instrumentation system that will measure the combined quadrature response. The author points out that this alone will markedly improve the measured mode shapes. However, when the modes have resonant frequencies close together, additional improvement can be obtained by separating the individual quadrature responses from the combined quadrature response. This is done by using a coupling matrix, derived in Reference 4, which is dependent only on the resonant frequencies and modal damping coefficients. When this matrix is inverted and post-multiplied by the matrix of measured mode shapes, the individual natural mode shapes are obtained.

A brief comparison of these methods is presented in Reference 6. The polar plot method is felt to be theoretically accurate, while other methods, such as peak amplitude, maximum quadrature component, and quadrature response, can be shown to be inexact. However, this particular quadrature response method is only the first part of the phase separation technique outlined previously. The separation of the combined quadrature into individual modal quadrature responses is the step that makes the phase separation technique as theoretically exact as the polar plot method. Both methods will suffer from experimental errors and instrument inaccuracies. The phase of the response with respect to the input force as well as amplitude must be measured for either method. The chief criterion on which to base a selection of either method appears to be the size of the data reduction task associated with each method. As pointed out before, polar plots would be necessary for each pickup location in each mode. If these plots are constructed manually, it is obvious that the task may take many weeks. On the other hand, the phase separation technique may take a week or two. This large time disparity could be eliminated by recording the data on magnetic tape and utilizing an analog computer, a digital computer, or a combination of both to construct the polar plots and perhaps even analyze these plots. Therefore it is reasonable to conclude that if the data are to be reduced manually, the phase separation technique should be used; if a computer can be utilized for the polar plots, there are no obvious advantages for either system.

If a multiple exciter system was used in the vibration test, it should have been possible to tune the system until the response in modes other than the mode of interest was quite small. Thus the data obtained should be similar to those obtained for a simple structure, and the same data reduction method can be employed. However, it would be wise to use one of the other methods at a few points to check the purity of the measured modes.

3.5.3 COMPUTERIZED DATA REDUCTION TECHNIQUES. As mentioned previously, if the data are recorded on magnetic tape, it is possible to analyze them on analog or digital computers. This is particularly effective for a total response data reduction method such as that suggested for simple structures. In the case of the analog, it is possible to play the tape through an analog circuit designed to interpret the data at the instant of peak amplitude on a preselected accelerometer.

This type of system can also indicate where resonance occurs by producing a transmissibility plot. This requires a frequency sweep of constant rate, which for large systems should be slow due to the response time lag of a large-scale vehicle.

Using a digital computer to interpret vibration mode data is very straight forward. The data are digitized or sampled at a high rate (at least 40 times greater than the response frequency of the mode) and recorded on magnetic tape, which can be fed into a digital computer for analysis of the data. This type of system has the advantage that all data are sampled at the same instant of time; no interpolation of peak values need be made. Also, no manual reading of the data is required, eliminating one chance for human error.

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4/TEST PROGRAM

This area of the monograph will discuss the actual conduct of testing, i.e., methods used for locating and "tuning" or "peaking" modes to be investigated. First, however, a discussion of what modes should be investigated in association with solid and liquid boosters is presented as a basis for the determination of these modes.

4.1 APPROACH

The first step in a successful test program is the definition of the frequency range to be tested. This then helps in the selection of instrumentation and the exciter system. This information also designs the maximum acceptable rigid body suspension frequencies. After selection of the type of accelerometers to be used, they should be located along the length of the vehicle in sufficient quantity to adequately define the response shape of any of the anticipated modes.

Exciter excitation points are selected and the exciters are mounted to the support structure with provision for transmitting the exciter force into the vehicle.

All measurement instruments should be calibrated just prior to testing. This includes all accelerometers, linear transducers, visual display units, and recording systems. The visual display accelerometers and transducers are selected and tied into the display. Remember that the instruments selected for display can vary from mode to mode. A typical setup for location of modal accelerometers used for the Saturn IB dynamic tests for bending mode definition is shown in Figure 12.

Before applying any excitation force to the vehicle, check the orientation of the vehicle to make sure that it is clear of all stops and is properly supported by the suspension system.

If torsional response measurement is desired, the modal definition instrumentation must be oriented so as to define this motion. Referring to Figure 12, accelerometers 1 through 23 are adequately located along the vehicle. The sensitive axis of the units should be normal to the plane of the figure. Another set of accelerometers should be located at 180 degrees around the vehicle exactly opposite units 1 through 23 with their sensitive axis normal to the figure plane. This represents a typical setup for torsion mode definition.

Again, referring to Figure 12, accelerometers 1 through 23 could be used for modal definition of longitudinal modes if their sensitive axes were aligned parallel to the centerline of the vehicle.

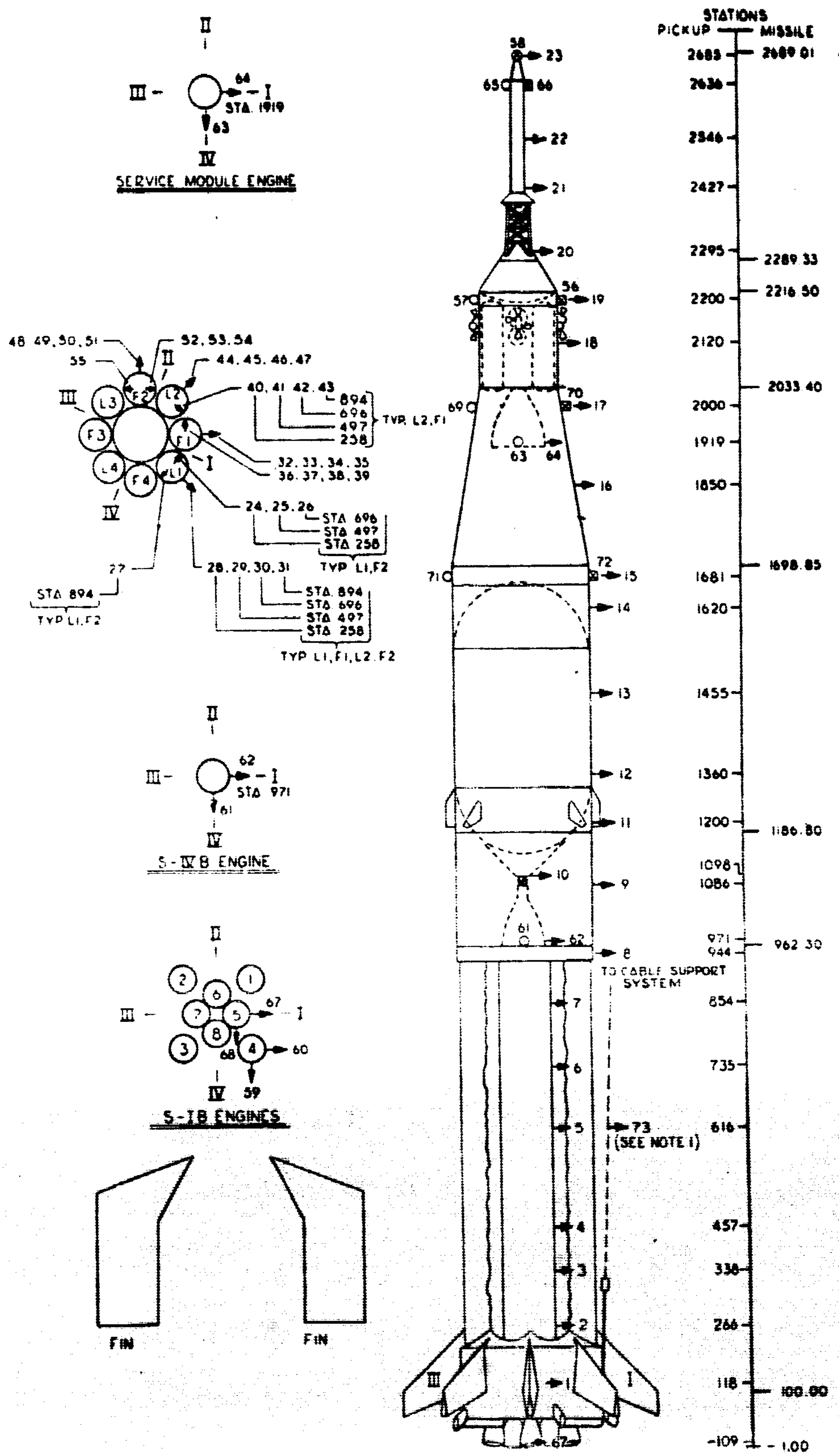


Figure 12. Accelerometer Location Drawing, SA-202 Configuration (Pitch)

4.2 PROCEDURE

4.2.1 MODES TO BE INVESTIGATED. The frequency range to be searched for elastic modes, propellant modes, and engine response modes is determined by the frequency response range of the control system. If, as an example, the control system maximum response frequency is 40 cps, then the test range is 0 to 40 cps.

After establishing the frequency range to be investigated, the analytically determined modes should be consulted as a guide to how many and what shape modes are anticipated in the test range.

4.2.1.1 Liquid Booster Modes. Of prime consideration in most vehicles is the first fuel sloshing mode, the first three elastic bending modes, and the engine first response mode. With these five basic modes as the prime objective, all other modes up to the control system cutoff frequency are of secondary nature but still of interest. For some boost vehicles like the Saturn IB that utilize clustered tank configurations, it may be any or a combination of the modes that exist in the control system response range that need prime consideration. A predetermined selection here of what modes are prime and secondary is quite difficult.

For large vehicles, the basic slosh mode will normally be below 1 cps. This is probably the most difficult mode to define since the vehicle response will display itself as a rigid body response. One method of determining that a fuel slosh mode is being excited is to mount a closed-circuit TV camera in the tank so that instantaneous visual monitoring of the liquid motion is available.

Pressurization of the liquid propellant tanks will not, in general, affect the modal response characteristics of the vehicle to any measurable degree after the tanks are pressurized to the point where tank shape is maintained. The effect of pressurization is a very small increase in tank diameter and a slight decrease in the height of the fluid in the tanks.

Cryogenic temperature effects may need consideration. At very low temperatures, the modulus of elasticity of the structure will be altered, although not greatly. If the program is to be conducted with actual fuels in the tanks, then the test results will reflect the cryogenic effects. If, however, the fuels are simulated with water or some other liquid, the analytical work must reflect this liquid and its temperature effects for comparison with the test results. Correlation of these results will then produce greater confidence in the analysis at cryogenic temperatures.

4.2.1.2 Solid Booster Modes. Again, the test frequency range is defined by the frequency response range of the control system. In solid propellant vehicles, the primary modes to consider are the first three elastic modes, the engine(s) response mode of the vehicle, and the first two elastic response modes of the propellant.

As is the case in liquid boosters, the propellant modes of the solid booster are the most difficult to define. An inert propellant does not display the proper visco-elastic properties to be used as a simulated fuel. This means that live propellant is required for propellant mode definition.

In instrumenting the live propellant, the accelerometers and their associated cables must be insulated against any voltage leakage or electrical spark. The accelerometer units should be mounted near the end of two or three adjacent star-points along the length of the propellant. Of course, it is realized that the defined modes are only representative for a short period of time since the surface burning of the propellant will alter its reponse characteristics continually through flight.

The other primary modes are not difficult to obtain. The engine modes can be relatively high in frequency due to their method of installation. Again all modes should be described in the frequency response range of the control system.

4.2.2 MODE DETERMINATION. This section presents the methods by which the modes of the vehicle may be located, tuned, and recorded. Distinction is made between three modes of interest at or near a resonant condition. These modes correspond to the following (where ζ is a dimensionless damping rate):

1. Resonant frequency (frequency of maximum amplitude): $\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$

2. Damped natural frequency (frequency at which a free vibration will decay):

   $\omega_d = \omega_n\sqrt{1 - \zeta^2}$

3. Natural frequency (frequency of the undamped mode that corresponds to the calculated mode and frequency): ω_n

These three frequencies are manifest in ascending order at each resonant condition. When the damping is small the frequency differences are hardly distinguishable. However, in tuning the shakers it is well to recognize which of these modes is being excited for there are significant differences in the modal characteristics for each. It is the damped frequency that should be sought in tuning the mode for this is the mode of minimum quadrature and the mode at which the free vibration will decay.

4.2.2.1 **Frequency Sweeps.** The classical method of locating vehicle responses is frequency sweep. This method is done by either automatic or manual control of the exciter system oscillator. To start, the oscillator frequency is set below the first anticipated elastic response frequency. Care must be taken not to set the frequency low enough to excite the rigid body modes of the vehicle/suspension system combination.

If automatic frequency sweep rate is used, the exciter force is set at a low value and the frequency is slowly increased through the test range. By continuous recording of the output of selected accelerometers, frequency locations of modal response will be indicated by peaks of the outputs of the accelerometers. The frequency should be swept again starting at the top frequency and, using the same sweep rate in reverse, going down to the original start frequency. This is done to bracket the true response since in using a constant sweep rate a vehicle response lag is inherent. By sweeping both directions two peaks will be indicated for each response mode at slightly different frequencies. The true natural frequency of the mode is located between the two indicated peaks. Reference 10 provides some insight to the effect of sweep rate on the test results.

When using manual sweep, it is impossible to maintain a constant sweep rate. Therefore, starting below the first anticipated elastic response frequency, the frequency is slowly increased until a peaking of the accelerometer outputs is displayed. This frequency is then noted and the frequency increased again until all indicated peak frequencies have been determined through the test range. It is suggested that a second sweep, or search, be made to check that no peaks have been overlooked. If time permits, a sweep with another exciter location is desirable.

4.2.2.2 Mode Peaking or "Tuning". After the peak frequencies have been determined, the problem of exciting each of these modes properly is next. Select the exciter closest to the indicated peak amplitude area of the vehicle. Disengage all other exciter units. Set the frequency oscillator at the indicated peak frequency and apply enough force to show good response on the visual display systems. Now, carefully adjust the frequency until the amplitude-to-force ratio is a maximum. After attaining this condition, instantaneously remove the exciter force from the vehicle. This is accomplished by either interrupting the armature current on electromagnetic exciters or by declamping in the case of hydraulic exciters. When the free vehicle damps out, very carefully note the damped frequency. This is the true damped natural frequency for that mode of the system. Then excite the system at the indicated damped natural frequency and remove force again. This process is repeated until the wave shapes and amplitudes are repetitive from run to run. Another excellent criteria as an indicator of proper tuning of a mode is a very smooth decay of the vibration when excitation is removed.

For all the desired modes, the procedure described above is repeated and the final tuned modal data are recorded in both the forced and free condition. It is suggested that a minimum of three records be made of each natural mode so that the repeatability of the data can be confirmed.

Another tool used in tuning a mode is the Lissajous figure. By putting the shaker force signal on one axis of an oscilloscope and the velocity signal of one of the vehicle accelerometers, located near the shaker attachment point, on the other axis, a Lissajous display results. When a mode is properly tuned, the Lissajous display on the oscilloscope will be a straight line canted at some angle on the scope dependent upon the ratio of the amplitudes of the two signals. A discussion of Lissajous figures may be found in Reference 11. In Figure 13 are shown three possible Lissajous representations that could exist during the tuning of a mode. Sketch (a) shows an ideal situation; (b) is representative of a third harmonic excitation; and (c) shows a typical out-of-phase relationship.

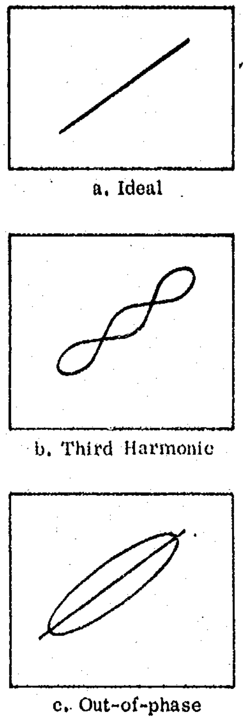


Figure 13. Mode Tuning Lissajous Figures

4.2.2.3 Modal Damping. Of equal importance with the shapes and frequencies of each mode is the equivalent viscous damping associated with each mode.

After a natural mode of the vehicle has been tuned and its shape recorded, the vibration force to the vehicle is interrupted and the mode allowed to damp out. Continual recording of the output of the accelerometers will indicate the equivalent viscous damping of the mode. This is very important in association with control system analysis as the analysis is conducted assuming a viscous damping. If the measured damping of the vehicle is less than the assumed damping, or even significantly larger, the analysis may need to be redone using the new damping factor to check vehicle stability.

5/DATA REDUCTION

In Section 3.4.2.2 a discussion was presented in connection with recording systems. Included was a short discussion of the output of these systems and how they are analyzed. Here, the subject of reduction of test data will be further expanded in the areas of quick-look and final data reduction.

5.1 QUICK-LOOK DATA REDUCTION

This type of data reducti is usually done on the spot during the test to ensure that the data taken are the desired data and that the test can be continued to the next test condition. This involves the use of the visual display systems.

When a mode has been tuned and recorded, the direct writing recorder data should be checked for smooth sinusoidal response, that no unusual or unexplainable frequency or phase shift is noted between the data traces, and that the mode is indeed the expected shape. By applying Equation 1 below, a quick indication of the system damping may be obtained.

$$\zeta = \frac{0.11}{N} \qquad (1)$$

where

ζ = dimensionless damping ratio

N = number of cycles to damp to half-amplitude

Damping ζ will usually have a value between 0.005 and 0.05.

Another quick method of deriving the approximate damping ratio may be obtained from the frequency sweep where the resonant frequencies are well separated so that each resonance may be considered as the response of a single-degree-of-freedom system. The method shown in Reference 13 is presented without proof. Observe the frequencies each side of the resonant peak where the amplitude is 70.7% of the resonant amplitude. Then the damping ratio for small values is given by

$$\zeta \approx \frac{\omega_2^2 - \omega_1^2}{4\,\omega_r^2} \approx \frac{\omega_2 - \omega_1}{2\,\omega_r} \qquad (1A)$$

where ω_2 is the frequency above resonance where the amplitude is 70.7% of the maximum amplitude, ω_1 is the corresponding 70.7% amplitude frequency below resonance, and ω_r is the resonant frequency (frequency of maximum amplitude).

The main function of this type of data reduction is to verify on the spot that the mode under investigation has been sufficiently defined and that no further modal resolution is required.

5.2 FINAL DATA REDUCTION

The method of accomplishing final data reduction is dependent upon the type of recording system used. Since some of these details have been discussed in a previous section, discussion here will be held to methods of reduction.

5.2.1 MANUAL DATA REDUCTION. The most difficult problem here is proper definition of the elastic mode shapes. The form of the raw data will vary depending upon the method of data reduction to be employed. For the simple method, the data will usually be in the form of the wave shape responses of the accelerometers, displayed with several channels of information per record. It may take several records to cover the complete instrumentation of the vehicle, so choosing a timing mark common to all records as a reference point will ensure that all records are read at the same instant of time. The time slice for reading will be that time when the highest-amplitude accelerometer is peaked. All channels of information are read at this time instant (not at their peaks) with some type of scaled reader and the values recorded and subsequently plotted. Reading all channels at the same instant of time lessens the effect of the out-of-phase contributions from other modes.

If the phase separation technique is to be used, the data will be simply a record of a signal proportional to the amplitude of the combined quadrature component of the response at a given pickup location. These data can be normalized and plotted directly, or they can be further refined using the coupling matrix mentioned previously and discussed in detail in Reference 4.

The data for the vector analysis method (Reference 3) are records of either quadrature and in-phase response or amplitude and phase response. Either set of responses will be as a function of frequency, and the total response is plotted as a vector at even intervals of frequency. A separate plot should be made for each mode at each pickup location. The point of maximum frequency spacing must be found; it may be necessary to take vector derivatives of the response vectors to accurately determine this point. A circle of curvature must be fitted to the plot at this point to determine the modal amplitude. Care must be taken in all plotting and determination of the maximum frequency spacings and circles of curvature.

The response frequency is determined by counting the number of cycles of response in a time period, then reducing this to a per second base. Frequency checks should be made at various locations along the vehicle, especially such parts as engine bells, to ascertain if all portions of the vehicle are responding at the same frequency.

Aggregate system damping (the ability of a system to dissipate energy) is of prime interest in control system design and is seldom predictable through the use of analytical techniques. Damping depends, almost exclusively for definition, upon test data from the vehicle. The data from which the information is gathered normally consist of a fixed point in space recorded in a grid in real time and will include the following effects:

a. Shift in zero reference.

b. Different scale factors each side of zero, including nonlinearity of same.

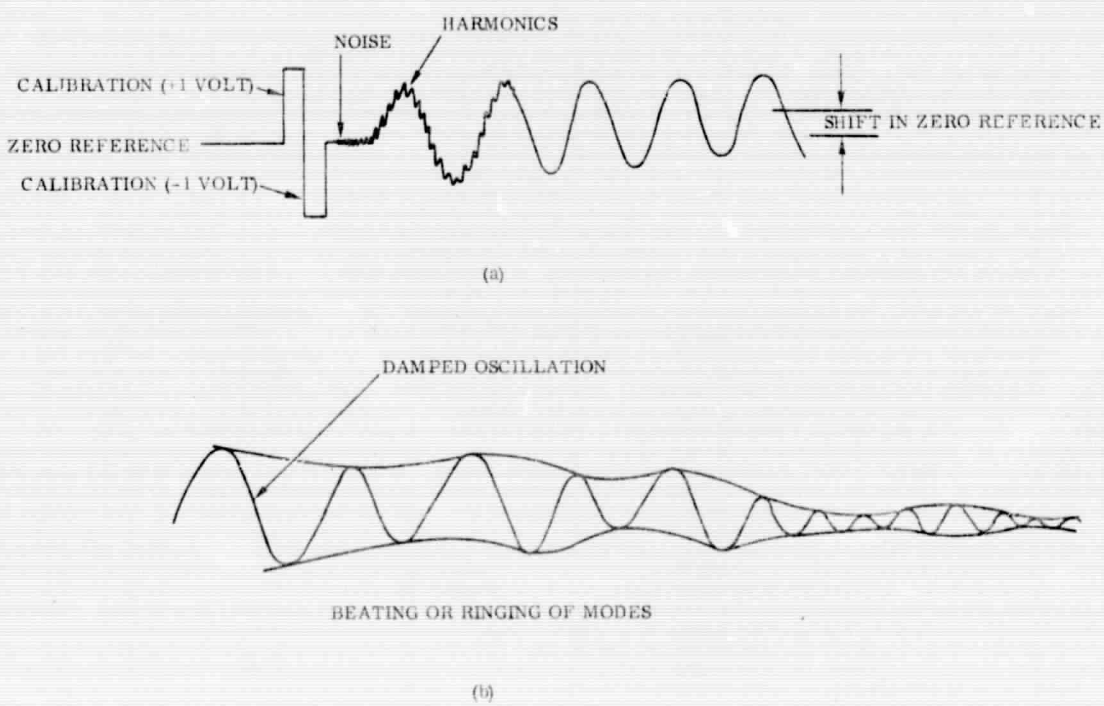
c. Nonlinearities in the system under study.

d. Harmonics.

e. "Beating" and "ringing".

f. Noise.

Examples of these effects are sketched in Figure 14. The degree to which these effects exist and their impact upon the method of data reduction determines the usability of the data.



NOTE: ALL OR ANY COMBINATION OF THE EFFECTS SHOWN IN (a) AND (b) ABOVE COULD BE PRESENT IN ANY PRESENTATION OF RAW DATA.

Figure 14. Examples of Superimposed Effects on Damping Determination

A number of equations can be derived, similar to the one shown in the previous section, based on the assumption of constant damping and measurements made over some arbitrary interval of time. These methods are only approximate but are useful to determine a quick estimate of the damping. A more refined and accurate approach is described in Reference 12 where the damping parameter is obtained from an overlay used in conjunction with semilog graphs of peak-to-peak amplitudes plotted against cycle number. The chief difficulty encountered in using this technique lies in the necessity of constructing the overlay or, alternately, the awkwardness of applying the damping equation (on which the overlay is based) directly.

The points that can be most accurately located visually on an experimental trace are the successive maxima and minima, except where excessive "clipping" or extraneous modes and harmonics are present. Such effects, if of higher frequency, will normally damp out quickly, causing the loss of only a few bits of information. The successive points of maxima (or minima) can be connected with straight lines as an approximation to the actual decay envelope with negligible error.

As an example, assume an oscillatory system with constant damping expressible as:

$$f(t) = A e^{-\zeta \omega_n t} \sin (\omega_d t + \phi) \tag{2}$$

where

A = waveform half-amplitude

ζ = dimensionless damping ratio

ω_n = natural circular frequency (rad/sec)

ω_d = damped circular frequency (rad/sec)

ϕ = phase angle (rad)

t = time (sec)

The double amplitude of the waveform is

$$g(t) = 2Ae^{-\zeta \omega_n t} \tag{3}$$

Transforming the ordinate to a logarithmic scale,

$$p(t) = \text{Log}_{10} g(t) = \frac{1}{2.3} \ln g(t)$$

$$= \frac{1}{2.3} \ln \left(2Ae^{-\zeta \omega_n t} \right)$$

$$= \frac{1}{2.3} \left[\ln 2A + \ln e^{-\zeta \omega_n t} \right]$$

$$p(t) = \frac{\ln 2A}{2.3} - \frac{\zeta \omega_n t}{2.3} \qquad (4)$$

which is geometrically linear with a slope of

$$\frac{dp(t)}{dt} = -\frac{\zeta \omega_n}{2.3} \qquad (5)$$

For small damping ($\omega_n \approx \omega_d$),

$$\zeta = \frac{2.3}{\omega_d} \frac{dp(t)}{dt} \qquad (6)$$

Replacing the derivative with an incremental approximation,

$$\zeta = -\frac{2.3}{\omega_d} \frac{\Delta p(t)}{\Delta t}$$

$$= -\frac{2.3}{2\pi} \frac{\Delta p(t)}{f \Delta t}$$

$$= -\frac{2.3}{2\pi} \frac{\Delta p(t)}{\Delta N K} \qquad (7)$$

since f is always observed to be constant. Therefore, the dimensionless damping ratio is directly proportional to the slope of the semilog envelope plot and is

$$\zeta(\%) = \frac{36.6}{K}\,\frac{\Delta Y}{\Delta N} \tag{8}$$

where

K = scale factor, linear units per logarithmic cycle used in the construction of the semilog plots.

$\frac{\Delta Y}{\Delta N}$ = slope of auxiliary plot, linear units per cycle.

Another method of computing damping is given by Equation 9 below.

$$\zeta = \bar{\zeta} - \frac{1.6445}{M} \sum^{M} (\bar{\zeta} - \zeta_i)^2 \tag{9}$$

where

$$\bar{\zeta} = \frac{1}{M} \sum^{M} \zeta_i$$

$$\zeta_i = \frac{2}{\pi}\left(\frac{F_{i-1} - F_{i+1}}{F_{i-1} + 2F_i + F_{i+1}}\right)$$

ζ = dimensionless damping ratio

M = number of peaks observed

$F_{i-1}, F_i, F_{i+1}, \ldots$ = amplitudes as shown in Figure 15.

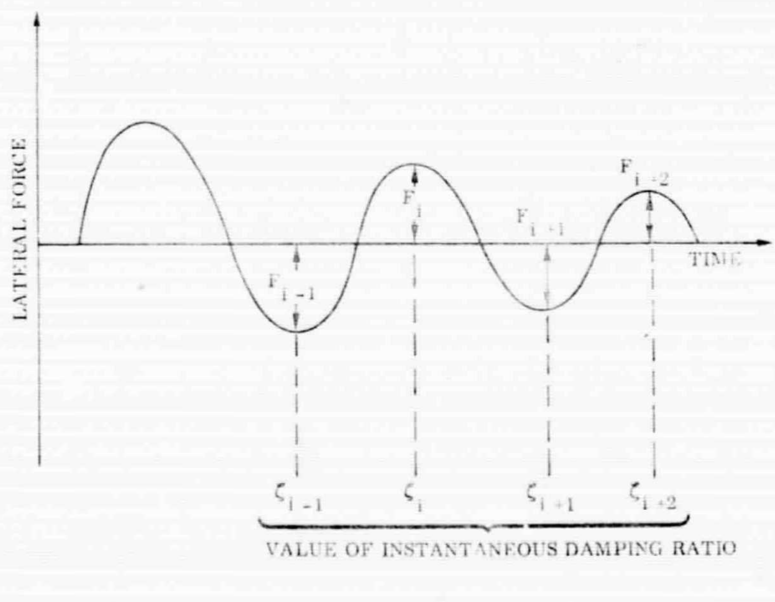


Figure 15. Damping Computation Amplitudes

Still another method for estimating damping is shown in Figure 16. To use this graph, determine the number of peaks required for a particular displacement ratio. Then, locate the point corresponding to the intersection of the amplitude ratio and the line representing the number of peaks. The damping ratio is then read off the abscissa.

When multiple shakers are employed so that their responses are in phase or 180 degrees out of phase, the damping ratio may be obtained from energy considerations by equating the shaker energy input to the energy dissipated at resonance. Then

$$\zeta \approx \frac{\Sigma\, Fx}{2\omega_i^2 \;\Sigma\, Mx^2}$$

where $\Sigma\,Fx$ is the generalized force input in phase with velocity of both the shakers and the reaction devices, and $\Sigma\,Mx^2$ is the generalized mass of the test article for the mode at frequency ω_i.

5.2.2 SEMI-AUTOMATIC DATA REDUCTION. When magnetic tape units are used as a primary data recorder, it is possible to play back the information into an analog computer properly programmed to partially reduce the data. This is only possible if a constant frequency sweep rate is employed in the test. This is used as a triggering device by the computer. The computer will reproduce the taped signals on an X-Y plotter and provide a simpler base for the manual reduction of the data.

In most cases determination of equivalent viscous damping still has to be accomplished manually as was described in the previous section. Some automatic methods can be mechanized in association with an analog computer, but these methods, at this time, are quite complex.

5.2.3 AUTOMATIC DATA REDUCTION. Another possibility for reducing modal data from a magnetic tape record is to record the data in digital form on the tape and then play it back through a digital computer program to interpret the information. The computer has to show which channel it will use as a reference. This is given to it as the accelerometer showing the largest peak amplitude. The computer will then read all channels of information at the instant the reference accelerometer indicates maximum amplitude. This reduced data will then be transferred to another tape which in turn will be plotted automatically by another computer.

It is also possible to have the positive and negative peak values of several consecutive damped waves read and plotted for purposes of determining the damping rate of the modes.

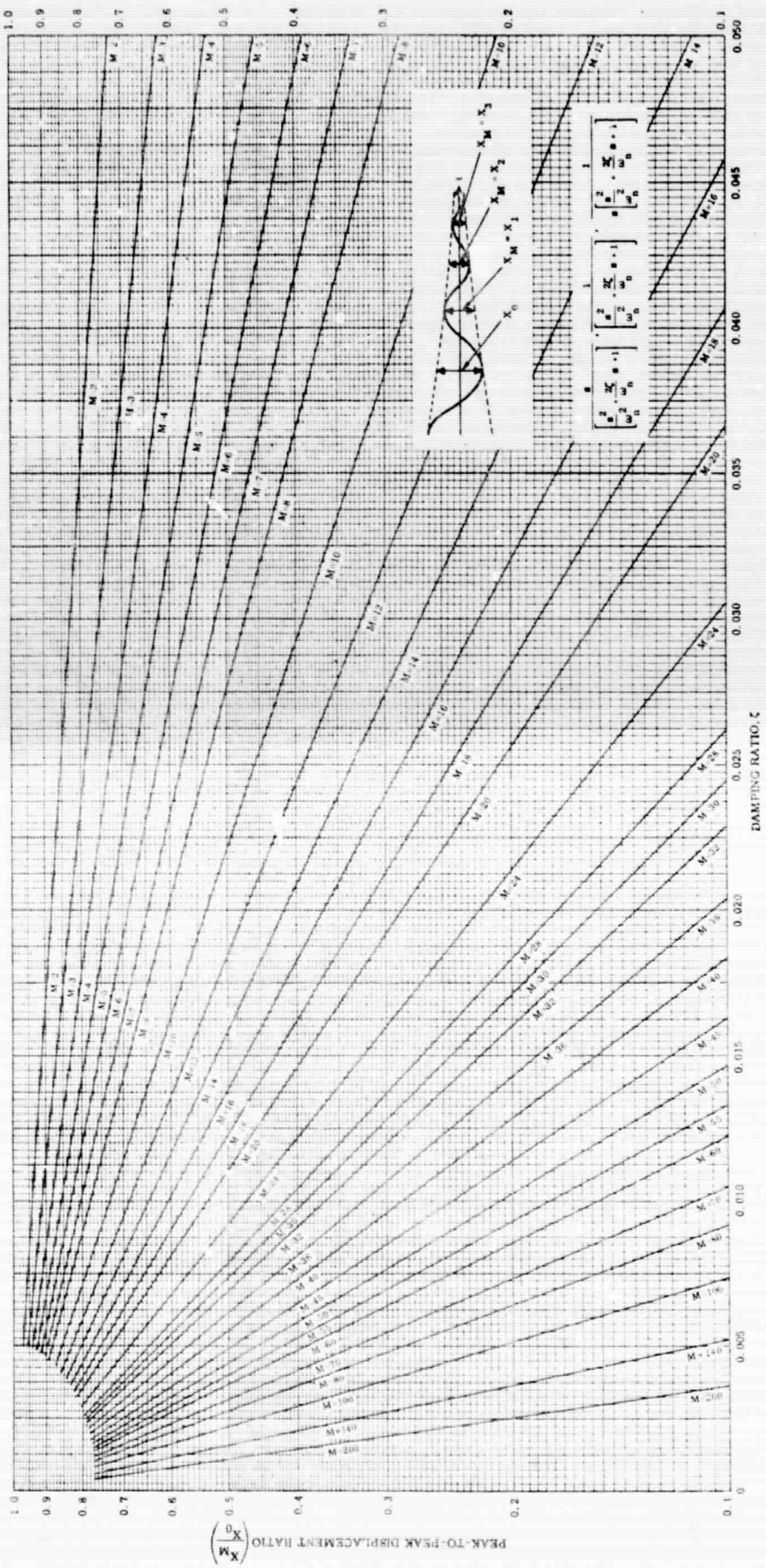


Figure 16. Method for Estimating Damping

6/PROBLEM AREAS

The presentation of this section will describe the possible areas where trouble might occur. These areas will cover everything from problems that affect data to problems that could mean disaster to a test program.

6.1 TEST SETUP

The problem areas discussed below are those arising from test conception to actual testing.

Early in the program, a type of suspension system, the support structure, and a method of mounting the excitation system are selected. After selection, each of these systems should be analyzed to determine its natural response frequency. This is done so that the final design of any of these systems will permit a minimal coupling effect with the test modes or cross-coupling between the systems themselves.

Coupling between the support structure and the suspension system will mean that any response of the support system will cause the vehicle to respond in rigid body motion. This motion will affect the elastic data to some degree, depending on how much motion is fed into the vehicle. If the support structure motion is large enough, vehicle safety might be impaired. As with any system with feedback, instabilities can occur.

The exciter mounting system should be decoupled from the support structure motion. Coupling of these systems could result in exciter damage and possible vehicle damage. Decoupling of these motions can be accomplished by isolating the exciter mounts with some type of mitigation system (e.g., bearing-mounted pendulous masses) that will not conduct any appreciable amount of the support structure motion into the exciters.

Close attention must be paid to the possible coupling of the suspended vehicle rigid body responses and the slosh mode. For vehicles of the size covered in this monograph, coupling of this type is a distinct possibility since the natural frequencies of both these responses will probably be below 0.5 cps. When determining the frequency and response shape of the rigid body modes, it is suggested that very low amplitudes be used if the modes are shaker excited. By using small amplitudes, little motion is imparted ɔ the fluid mass. If the fluid mass is started in motion, the energy exchange between the rigid body response and the slosh mode can prove to be divergent.

It is possible to "hand-excite" the rigid body modes if shaker excitation is not possible. In doing this, the vehicle is simply hand forced and allowed to damp at its rigid body frequencies. Again, do not impose high amplitudes when exciting these modes.

As mentioned in the previous section, pressurization above a 15-psi level does not appreciably affect the frequencies and shapes of the elastic modes. However, high pressures (up to 70 psi) can be required to preclude a structural failure such as bulkhead reversal of a fully tanked vehicle. Safety of test personnel is the main consideration in connection with high pressures. The amount of fuel ullage area in a tank at a high pressure is directly associated with the degree of explosive energy available if a leak or tear should show up in the tank.

The coupling of solid fuel modes and suspension system modes does not have the possibly disastrous side effects of the liquid-fuel vehicle because of the high damping of the solid fuel. The effect manifests itself mainly as a distortion of data and the resultant mode shape.

Analytical prediction of the response frequency of viscoelastic material (such as solid propellant) is less than perfect. To offset this deficiency, adjustment should be built into the suspension system so that the suspension frequency can be altered for decoupling of the fuel and suspension responses.

When considering safety systems, first protect the test article and secondly protect the test equipment. Test article protection has been discussed under Section 3, so emphasis here will be on equipment protection.

If electromagnetic exciters are used some type of low-yield member must be included in the attachment between exciter and test article. This unit is designed to fail if misalignment or force outside the tolerances of the exciter unit occurs. One type of unit often used is the "fuse" sketched in Figure 17. Because of the necked-down area, the fuse will break if misalignment or overload occurs. If hydraulic excitation is used, protection of both the exciter and test article can be assured by putting shear pins in the attachment between the exciter and the test article. These shear pins will yield if forces of too high a magnitude are experienced.

6.2 TEST PROGRAM

Discussed first under this heading are two basic problems with hydraulic excitation units.

First, testing and tuning with this type of unit is slower and more demanding because of the response time lag of hydraulic units. After a small change in force or frequency is made, there is a slight delay before the change is noted in the response

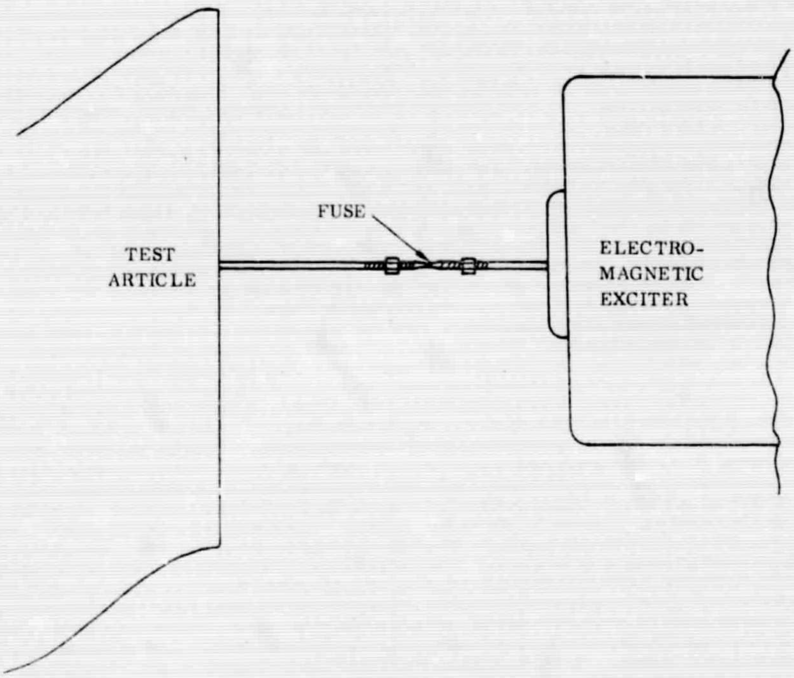


Figure 17. Typical Electromagnetic Fuse Attachment

of the vehicle. Associated with this is the difficulty of phasing two or more hydraulic exciters to show in-phase or 180-degree out-of-phase relationships.

The second major problem in using a hydraulic excitation system is the difficulty of maintaining "clean" output signals. This will affect data, tuning of modes, and maintenance of constant force or displacement levels. Displacement, velocity, and/or force feedback loops within the exciter systems go a long way in producing acceptable signals and are strongly suggested for incorporation in any hydraulic exciter design.

If only bending responses of the vehicle are desired, the accelerometer units should be oriented so that measurements are made in the plane of excitation only. Even though accelerometers that are sensitive in one direction only are used, response to motions in the other planes will be picked up to some degree. By taking care that the accelerometers are oriented with their sensitive axis directly in line with the plane of excitation and making sure that no canting in the vertical plane of the vehicle is apparent, the "cross talk" or off-axis response will be kept to a minimum.

The accuracy of the data is very sensitive to the quality of the measuring equipment. Even the best of amplifiers and recording equipment will have errors of about 3 percent of full scale. Long landlines connecting the measuring equipment with the measuring instruments will also induce inaccuracies. All these effects will appear as distortions of the data.

By selecting accelerometers with nearly identical response characteristics, landlines of the same length, and matched amplifiers, the inaccuracy effects can be kept to a minimum.

A major difficulty that can be encountered during testing is coupling of elastic modes. Under certain conditions two elastic modes will occur at frequencies very close to each other. This will make it difficult to excite one mode without seeing considerable response from the other mode. Separation of these modes may be accomplished by using one of the other exciters or a combination of exciters and adjusting their force levels and phasing until one of the modes is properly driven. This, along with fine frequency tuning, should separate the two modes with good definition of each.

Coupling of the vehicle elastic and rigid body motion is always in evidence to some degree when testing in a free-free condition. By designing the suspension system to display rigid body frequencies well below the first elastic mode of the vehicle, this effect can be kept to a minimum. Eliminating this effect from the data will be covered in Section 6.3 below.

6.3 DATA REDUCTION

This area is probably the place in the test program where more errors occur than any other. This section will attempt to point out some of the problems that occur and how they may be circumvented or minimized.

6.3.1 GETTING TRUE MODE SHAPES OUT OF THE DATA. The most common error made here is reading the peak-to-peak or zero-to-peak data on all recorded channels of information. As the vehicle responds in its natural modes, the response of all points recorded along the vehicle are not exactly in phase, 180-degree phase, or 90-degree phase. Some of the points will display slight phase shifts (usually less than 10 degrees). The true mode shape can be obtained from slight out-of-phase data by reading the amplitude of the data on all channels at exactly the same time slice. However, if these phase shifts become appreciable, a data reduction method such as those already discussed should be employed. Being able to know in advance whether or not the phase shifts will be appreciable requires someone with vast experience and good intuition. Therefore, a good rule of practice would be to expect the phase shifts to be significant and use an instrumentation system that will measure both amplitude and phase. If the phase shifts turn out to be negligible, one has the data to justify the simple data reduction method.

Another problem affecting the modal information extracted from the recorded data is the superposition of rigid body motion on the modal data. Elimination of this effect can be accomplished by the method explained in Appendix I of Reference 14.

6.3.2 MODAL BEATING. The phenomenon of modal beating usually appears when the vehicle being tested is a multiple-beam structure such as a clustered tank configuration, multiple engine bells, or payloads housed within fairings. Modal beating occurs when two structural members respond at frequencies very close to each other and there is an energy exchange between them. The data will indicate one of these members responding with the other relatively quiet. Then, through the exchange of energy, the quiet member will pick up amplitude while the other member will lose amplitude. This condition will continue until the motion of both members has damped out. In theory this beating should decay in time, and by fine tuning the modes should be separable. In practice, however, each of the modes will exhibit some amplitude dependence or other nonlinear phenomena which will make separation impossible.

When this effect appears on the data, it is suggested that the mode shape be read at two or more points in time to cover the maximum amplitude conditions of both members.

6.3.3 GETTING TRUE MODAL DAMPING. The pitfalls associated with this subject have been covered in Section 5.2.

The emphasis here is that quick methods of getting the modal damping (such as Equation 1) are adequate for quick-look information but do have approximations associated with them. A representation of the true modal damping can only be gotten by applying more meticulous methods such as those covered in Section 5.2 and then only if the damping mechanism fits the classical mathematical model.

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7/SCHEDULE PREDICTION FOR A LARGE-SCALE DYNAMIC MEASUREMENT PROGRAM

This section presents a typical schedule for a dynamic test program suitable for obtaining all experimental data necessary for the design of a vehicle control system. The times given are averages based on past tests conducted both in industry and in government facilities; a wide variation in these times can be expected depending on the level of effort directed toward the testing.

7.1 PRE-TEST

Covered in the pre-test phase is the time from the conception of the program to the actual testing. This discussion assumes the prior existence of a test article and test stand, therefore allotting no time for the construction of either.

The first step, after the need of a vibration measurement program has been established, and the availability period of the test article is determined, is a meeting of all contributing areas to establish test requirements and to define responsibilities. After a meeting of this sort, a preliminary test plan should be prepared that should cover as many phases of the program as can be well defined at this stage. The writing, reproduction, and distribution of this plan will take approximately a month.

If no suitable suspension system is available, the next major effort will be the design, construction, and fabrication of such a system. Approximately three to six months is required, depending on the type and complexity of the suspension system selected.

Concurrent with the design of the suspension system is the selection, construction, and checkout of the instrumentation and recording equipment (again, assuming this equipment is not available at this time). This should be completed easily within the period allowed for the suspension system completion. Also in this time period the design, construction, and fabrication of the safety equipment; calibration and mounting of all transducers; design, construction, and installation of stabilizing systems; and the writing of the final test plan should take place. If the suspension and measuring system are available for the test this time period can be reduced to one to three months.

When all systems are designed, constructed, and installed, another month should be allowed for checkout of the completed test setup. This amounts to a test setup time of from five to eight months. A nominal figure for this portion of the schedule is six months.

7.2 TEST

The ability to predict the time required to run a given test program is full of pitfalls. The test time required is a function of a variety of things, including the experience of the manpower available, the condition of the equipment being used, the weather, random failures of equipment and vehicle, the amount of equipment assigned to the program, and the priority assigned.

An example how this might vary can be shown by two separate full-scale mode programs carried out on the Atlas/Agena/OAO configuration and the Saturn IB programs. In the OAO program a normal priority was assigned and manpower was based upon a 40-hour work week. For this program it turned out that a fairly good rule of thumb was one to two weeks of test time for each fuel condition with a changeover time between test configurations of at least 2 days; thus for this type of program involving four fuel conditions a test time of 5 to 9 weeks would be required.

For the Saturn IB program, which was based upon a maximum effort, the above times can be reduced tremendously. It was possible for Marshall Space Flight Center to conduct a complete test for a given fuel condition in one direction of excitation in less than one day. Under actual operation the Saturn IB was dynamically tested for 2 fuel conditions in pitch and 3 fuel conditions in roll in five days. An example of how this was accomplished can be obtained by reviewing the times given below for running a particular fuel condition, including the time required to change planes and fuel condition.

TIME	TESTING
0.5 day	Run fuel condition
1.0 day	Changeover instrumentation from one plane to another
0.5 day	Change suspension system
0.5 day	Change fuel condition

Under an efficient all-out program such as conducted by MSFC, the number of shakers available was such that no shakers had to be moved. In addition, some of the tasks can be overlapped such as changing fuel condition and suspension system simultaneously. Since more than one fuel condition is run in any given plane, one can readily see that with efficient management an average of a fuel condition per day is possible.

7.3 POST-TEST

This time period covers the time from the completion of the test to the publishing of the final report.

Data reduction time for the test program discussed in the above section will consume about one week for each week of testing. This means nominally six weeks of data reduction. The analyzing and plotting of these data will take another month.

The assimilation of this information into a final report and publishing will take approximately two months. This results in a test program duration of approximately one year plus the actual testing time from test conception to the publishing of a final test report.

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8/QUICK TEST METHODS

Very often the time for a major test program is not available or the need for a comprehensive test program is not present. In these instances, shorter, less informative test programs are undertaken.

Some types of investigations that do not require major test programs are:

a. Gyro response to the elastic modes.

b. Gimbaled engine response to the elastic modes.

c. Modal response in areas where delicate electronic packages are to be mounted.

8.1 APPROACH

The major concern here is where to conduct a test of this type. Possibly a suspension system and tower are available and can be utilized for testing. If not, some type of transporter for the system (e.g., crane, hoist, trailer) is available and might be used as a test bed. The launch pad itself is a strong possibility. Any one of these systems might be utilized as a test bed and along with a good analysis a reasonably limited information program can be conducted.

Instrumentation requirements for a quick-test program must be kept to a minimum. The control system gyros themselves act as the measuring instruments in the first test case cited above. The only additional measurement instruments required are some type of recording equipment to monitor the gyro outputs.

In the second case, three or four accelerometers located from the gimbal point to the base of the bell will define the engine response. A small number of transducers placed over the vehicle length will adequately define the vehicle body response. Some portable amplifiers are needed along with recording equipment to monitor the outputs of the transducers.

Accelerometers located in the area where the electronic equipment is to be mounted will define the response characteristics of this area. Again, portable amplifiers and recording equipment have to be supplied for monitoring the information.

Excitation can be supplied by electromagnetic exciters in the 50 to 100 pounds-force range. In the first and third cases, excitation can be defined at or near the nose of the vehicle. In the case of engine bell excitation, force may be applied at the lower portion of the engine with an exciter in the range of 25 pounds-force.

8.2 LIMITATIONS

Because of the simplicity of a program of this magnitude, complete modal information cannot be obtained. Definitive data are obtained from localized areas with minimal data available over the remainder of the vehicle.

Limited control of excitation units is inherent in this type of program, thereby requiring attentive handling of the excitation equipment. These tests are usually performed on a flight article or a vehicle designated for another prime program, thus requiring little or no alteration to the vehicle. Care must be used in exciting the vehicle to ensure against damage.

Because of the type of instrumentation and recording methods used on a quick-information test program, accuracy of the resulting data will be reduced.

9/SUMMARY

This monograph has attempted to present the methods by which a vibration measurement program on a large, full-scale boost vehicle might be successfully conducted. Coverage of the program from test conception through the final test report is presented.

In all areas of the monograph, various methods of accomplishing each individual task have been presented along with advantages and disadvantages of each method.

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